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PRICING LIFE ANNUITIES FOR IMPAIRED LIVES: THE CASE OF PORTUGAL

PRECIO DE LAS RENTAS VITALICIAS PARA PERSONAS CON ENFERMEDAD: EL CASO DE PORTUGAL

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Abstract

Life annuities markets are underdeveloped in Portugal and other countries. This annuitization puzzle is explained by improvements in mortality at old ages and passive adverse selection, reasons that put lives with diminished life expectancies at an unfair disadvantage. Using data from the SHARE (Survey of Health, Ageing and Retirement in Europe) project, we assess the impact of some of the most serious and common diseases over a reference survival curve. Then we calculate the price of annuities for impaired lives, using adjusted survival, and compare them with those for the reference population. We show that applying the reference mortality to impaired lives is very unfair and that pricing annuities for lives weakened by disease in an accurate way is possible.

Keywords: annuities, impaired life, net survival, crude survival, SHARE.

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Resumen

Los mercados de rentas vitalicias están subdesarrollados en Portugal y otros países. Este "*annuitization puzle"* se explica por las mejoras en la mortalidad en edades avanzadas y la selección adversa pasiva, razones que ponen en una desventaja injusta a las personas con una esperanza de vida disminuida. Usando datos del proyecto SHARE (*Survey of Health, Aging and Retirement in Europe*), evaluamos el impacto de algunas de las enfermedades más graves y comunes sobre una curva de supervivencia de referencia. Luego calculamos el precio de las rentas vitalicias por deterioro de la vida, utilizando la supervivencia ajustada, y las comparamos con las de la población de referencia. Mostramos que aplicar la mortalidad de referencia a las vidas deterioradas es muy injusto y que es posible fijar el precio de las rentas vitalicias para las vidas debilitadas por la enfermedad de manera precisa.

Palabras clave: rentas vitalicias, vida deteriorada, supervivencia neta, supervivencia bruta, SHARE.

1. Introduction

In Portugal, as in other countries where private pensions are not mandatory, the annuity market is underdeveloped. Annual statistics from the Portuguese supervisor show that only a small proportion of individuals buy annuities, which correspond to less than 2% of the direct insurance production (see Autoridade de Supervisão de Seguros e Fundos de Pensões, 2019).

Since the individual's time of death is uncertain annuities should be a valuable product (cf Yaari, 1965). Evidence from several countries (cf James, Vittas and Song, 2001; Johnson, Burman and Kobes, 2004) shows the opposite. How could this annuity puzzle, as Modigliani (1986) named it, be explained?

The fact is more puzzling as the trend of life expectancy and the proliferation of Social Security reforms and Defined Contribution pension plans all over the world, Antolin *et al.* (2009), are expected *to increase* the demand for life annuities, since these are a straightforward way to hedge longevity risk.

There are two approaches to explain the annuity puzzle: the rational models framework and the behavioral economics framework. Under the rational models approach, the main explanations are: bequest motive, see Yaari (1965), Brown and Poterba (2000), Johnson, Burman and Kobes (2004),

Brown, Casey and Mitchell (2008) and Brown *et al.* (2008)); pre-existing annuitisation, see Dushi and Webb (2004); risk sharing in couples, see Brown and Poterba (2000); *and adverse selection*, which has a very strong impact on impaired lives and will be given more detail in the next paragraphs. The behavioral economics framework is beyond the scope of this work, see e.g. Brown (2007) or Benartzi, Previtero and Thaler (2011). It is an approach that attempts to explain the consumer behaviour based on psychology insights. The main goal is to incorporate some predictable consumer psychological biases that are not compatible with the hypothesis of pure rationality.

In Finkelstein and Poterba (2002) the main difference between the expected present value of all future annuity payments and the premium paid for the annuity reflects what they call active and passive selection. Active selection refers to adverse selection coming from private health information about expected longevity. Passive selection refers to factors that are also correlated with mortality of the annuitants. In particular, passive (adverse) selection refers to annuity prices being perceived as unfair for individuals with average or diminished life expectancy. Enhanced annuities should be available for such people since they are expected to receive payments for a shorter period of time.

Blake and Burrows (2001), Blake, Burrows and Orszag (2002), Brown and Orszag (2006), Blake, Cairns and Dowd (2006), and Friedberg and Webb (2007) conclude that the uncertainty about the health conditions of the annuitants makes it impossible for insurance companies to adequately hedge the longevity risk; therefore, they must charge a higher price to compensate for the risk taken. Brown (2001) and Brown, Casey and Mitchell (2008) show that only healthy individuals are willing to pay for an annuity product, which implies that the annuity market for individuals with poor self-reported health almost does not exist.

Since annuity prices being perceived as too expensive for individuals with average life expectancy, *and even more for impaired lives*, is one of the reasons for the puzzle (see Brown and Scahill (2010), Fong (2015), Gatzert and Klotzki (2016), Gatzert, Hoermann and Schmeiser (2012), Palmon and Spivak (2007), Steinorth (2012), and Webb (2006)), the objective of our study is to find ways to minimize this type of selection in annuity markets, regarding particular groups of weakened lives. To accomplish this, it is necessary to estimate the survival functions for such populations, using available data on the diagnosed medical condition, age at diagnosis, and time from diagnosis. The relevant event is the observation of an individual's death by the disease of interest, within the group of diseased people.

This paper has two parts. The first focuses on the mortality of impaired lives, an impaired live being someone who suffers from a pre-existing medical condition responsible for reducing his/her life expectancy. The second part performs a comparison of life annuity premiums for the reference population; this is the Portuguese population, *proxied* by GKM95 (male) and GKF95 (female) life tables, widely used by insurance companies in Portugal, and the premiums for the population of diseased individuals, using the mortality corrected by the factors estimated in the first part, for each age group and medical condition.

The following groups of medical conditions, which are the leading causes of death, are considered: cancer, cardiovascular diseases, namely heart attack, hypertension, cholesterol and stroke, and respiratory diseases, namely chronic lung disease and asthma. Data is supplied by the SHARE (Survey of Health, Ageing and Retirement in Europe) project, see Börsch-Supan *et al.* (2013) and Börsch-Supan and Gruber (2019). The text is structured as follows. Section 2 describes the methodology and Section 3 presents the data. The empirical study and results are in Section 4 and Section 5 concludes.

2. Methodology

2.1. Main Concepts

The existence of competing risks is one of the most important problems in the interpretation of the outcomes from survival analysis, since they can hinder the observation of the relevant event, or modify its probability. In the literature there are two different approaches to deal with competing risks, when estimating the survival function of a group of patients diagnosed with a particular disease: net and crude probabilities of death - see Berkson and Gage (1952), Cornfield (1957), Cutler and Ederer (1958), Ederer, Axtell and Cutler (1961), Chiang (1961), Wong (1977), Prentice *et al.* (1978), Berry (1979) and Dickman and Coviello (2015). Briefly, net probabilities of death are not influenced by changes in mortality from other causes, thus providing a useful measure for control of the disease of interest over time, and crude probabilities of death measure the mortality experienced by a cohort of people with the disease, thus estimating the risk of death from that disease when all causes of death are possible. Both crude and net probabilities can be estimated using cause-specific survival methods or relative survival methods.

The cause-specific methods estimate the probabilities of death from the disease under study and the probabilities of death from other causes, for a cohort of individuals diagnosed with the disease of interest, by using cause of death information. Erhardt (1958) and Spiegelman *et al.* (1958) show that the information on cause of death in certificates is inaccurate and incomplete. Percy and Dolman (1978) and Percy and Muir (1989) observed that the cause-specific survival approach is inappropriate for international comparisons since the coding practices vary substantially among countries, and Parkin and Khlat (1996) and Pineda, White, Kristal and Taylor (2001) show that when comparing cause-specific survival rates across diverse populations, if different racial or ethnic groups have different rates of follow-up, the estimates produced are not comparable. Accounting for the limitations of the cause-specific approach, relative survival methods, that are independent from potential miscoding of cause of death, are elected to report survival rates when international comparisons are made (Coleman *et al.* (2008)).

The concept of relative survival was introduced by Berkson (1942), who proposed an estimator for the net survival in cancer patients. It was later developed by Ederer *et al.* (1961). Relative survival is an approach to measure the excess mortality experienced by a group of people diagnosed with a certain disease, in comparison with a similar group of people in the general population. Relative survival models are largely applied in literature to study the excess of mortality due to cancer disease, but Nelson, Lambert, Squire and Jones. Nelson *et al.* (2008) show that the utility of this methodology is not restricted to cancer analyses and apply it to coronary heart disease.

The expected survival rate is usually available from general population life tables. However, life tables are not free of the specific disease under study, in the sense they reflect the force of mortality from all causes of death. Berkson (1942), Berkson and Gage (1950), Cutler, Griswold and Eisenberg (1957), Milmore (1958) and Ederer and Heise (1959) defend that the presence of the disease under study in the general population only produces a very negligible effect on relative survival estimates. Nelson *et al.* (2008) warn that this is a risky assumption, exemplifying with heart disease, which is the most important cause of death, essentially for oldest age groups.

Howlader *et al.* (2010) show that cause-specific survival may be preferable to the relative survival approach in the case of minority groups, for instance, minority racial subgroups, different socioeconomic strata and populations with strong risk factors for disease. The main problem of this approach is the difficulty to obtain complete and accurate information about the cause of death, which can lead to biased estimates. Since accurate and sufficient cause

of death information needed to apply the cause-specific method is not available, we will use only the relative survival methods.

2.2. Estimation Methods

The different methods can be characterized according to the desired measure (net or crude probabilities), the framework (cause-specific or relative survival), and the estimators defined, see Dickman and Coviello (2015) for a very clear discussion. In our study, as already explained, the framework is relative survival. In general, the *relative survival* framework provides a relative survival ratio r_t, defined as the ratio of the survival of the diseased people under study $\left(S\left(t\right)\right)$ to the expected survival of a comparable group of the background population assumed to be free of the disease $\left(S^*\left(t\right)\right)$,

$$r_t = \frac{S\left(t\right)}{S^*\left(t\right)}. \qquad (1)$$

While $S\left(t\right)$ includes the observed individuals diagnosed with the disease of interest, $S^*\left(t\right)$ is estimated from life tables, which means that not only healthy individuals are considered, but also those diagnosed with any kind of disease, including the disease of interest. Usually, this does not affect the estimates significantly, since the percentage of diseased individuals considered in the background population is negligible.

The most commonly used methods to estimate r_t under the relative survival framework were developed by Ederer *et al.* (1961) - known as Ederer I -, Ederer and Heise (1959) - Ederer II -, Hakulinen (1982) - Hakulinen -, Perme, Stare and Estève (2012) - Pohar Perme -, and Cronin and Feuer (2000) - Cronin and Feuer. The first four allow to compute net survival (and from now on will be associated to the calculation of the Net Relative Survival Ratios – NRS). The last one addresses the calculation of crude survival (and from now on will be associated to the calculation of the Crude Relative Survival - CRS).

Ederer I, Ederer II and Hakulinen are classic estimators and the estimated survival via each one of them is the ratio of estimated observed survival to estimated expected survival. The difference between the three comes from the way they estimate expected survival. Pohar Perme is more recent and aims to correct the fact that the classic estimators generally overestimate net survival, especially in the elderly. Cronin and Feuer propose a method to calculate the cumulative crude cause-specific probability of death for a population

diagnosed with cancer, in the presence of other causes. An essential description of the methods is in Appendix, where equations (2)-(12) are given. Further details can be found in the original works and also, for instance, in Pokhrel (2007).

3. Data

We use data obtained from the *easy*SHARE data set, see Gruber, Hunkler and Stuck. (2014) for methodological details. SHARE is a cross-national longitudinal panel database providing information on health, socio-economic status and social and family networks about people aged 50 or older, from 27 European countries, plus Israel. The *easy*SHARE release 7.0.0 is based on SHARE Waves 1 to 7 (all the details can be found searching the following DOIs: 10.6103/SHARE.w1.700; …w2.700; …; …w7.700).

The *easy*SHARE database is used because it stores information in one single dataset. It was necessary to include information about the type of disease, the start age of disease, for each participant, and the information about death conditions available since Wave 5 - end of life interviews. This way there is information on the death date, age at the moment of decease, the cause of death and the period of time the person had been ill before decease.

Our subsample includes males and females from Austria, Belgium, Czechia, Denmark, Estonia, France, Germany, Greece, Hungary, Ireland, Italy, Netherlands, Poland, Portugal, Slovenia, Spain, Sweden and Switzerland. Our aim being to compare the life expectancy of individuals suffering from a given condition with healthy individuals, if no restrictions are made over the time period of diagnosis, the results tend to be biased since, for those diagnosed a long time ago, we can just observe the survivors (we cannot observe failure time). To overcome this situation the data was restricted to individuals diagnosed with the disease of interest from 1985 onwards.

Another potential source of bias in the database is the high number of censored observations, since a considerable number of individuals drop out between inquiries. Censoring just leads to unbiased results if it is random and non-informative. There is evidence that data does not respect this assumption and mortality observations are underestimated. To deal with this problem data was changed in order to consider that a percentage of the observations that were not possible to follow-up in the following years have experienced an event of death, assumed to occur at the middle of the average time between interviews.

To test the results' sensitivity, all methods were estimated considering four different percentages: 0%, 40%, 80% and 100%.

From the four scenarios, only the third and fourth produced results in agreement with the literature in the sense that healthy individuals have a higher life expectancy than diseased ones. Hence it was considered the assumption of 80% mortality over censored observations, in order to take a more real and, simultaneously, more conservative hypothesis.

The sample gathers individuals from the listed countries. The countries with more responses are Belgium, France and Italy. Portugal is one of those with lowest weights, see Figure 1.

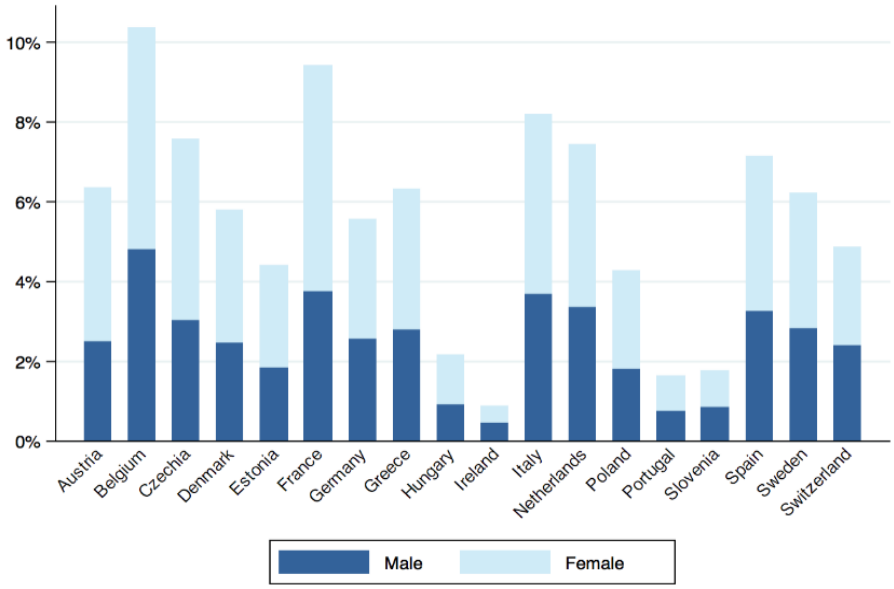


Figure 1. Percentage of males and females in the sample, by country.
Source: (a) Authors' calculations from SHARE database.

Considering the low weight of the Portuguese population, and being of interest to obtain results that can be applied to Portugal, a comparison was made of the distribution of deaths by cause in the database (Figure 2).

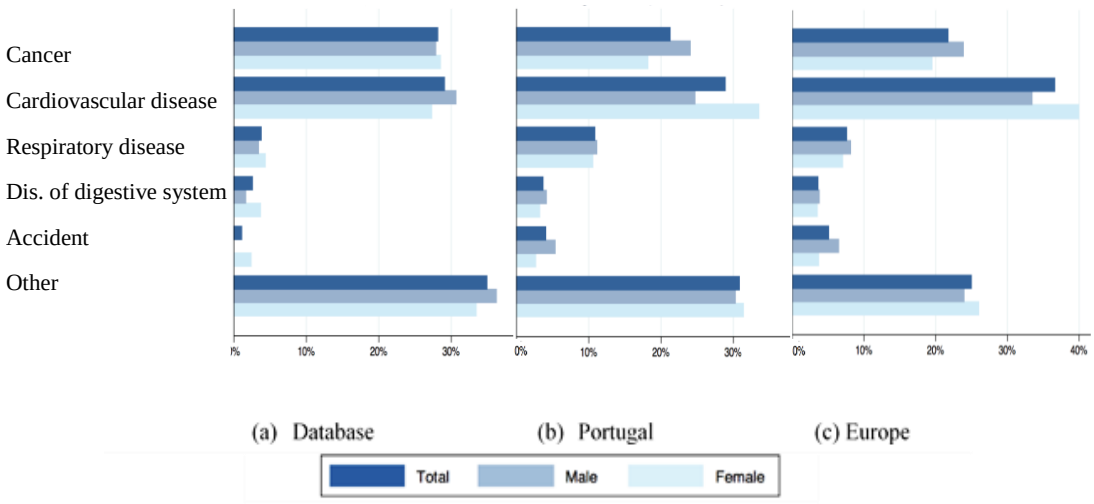


Figure 2. Percentage of deaths by cause. Source: (a) Authors' calculations from SHARE database; (b) and (c) Eurostat.

In spite of minor differences in the three graphs, a common pattern can be observed. Cardiovascular diseases, which include, among others, heart attack and stroke, are the main cause of death, followed by malign tumors. The percentage of death by respiratory disease is higher in the Portuguese population, which agrees with data from Eurostat that shows that Portugal is one of the countries with higher mortality rates by respiratory diseases.

This sample is very representative of the oldest population, being the age at diagnosis of three quarters of the total individuals greater than 50 years. The proportion of females is slightly higher than that of males, the majority of individuals are married, and have, on average, approximately two children. The mean level of education in the sample is nine years of schooling. About 23% of total individuals are employed, 43% are retired and the remaining 34% are unemployed or permanently sick or disabled.

The group of individuals under study was diagnosed, on average, with only one chronic disease, which allows to avoid competing risks. There is information about heart attack, hypertension, high blood cholesterol, stroke, diabetes, chronic lung disease, asthma, arthritis, osteoporosis, cancer, stomach or duodenal ulcer, peptic ulcer, Parkinson disease, cataracts and hip fracture or femoral fracture. However, some of these diseases are unrepresentative and were excluded from the analysis.

4. Application

4.1. Net and crude relative survival estimates

This section presents the effect of certain diseases on the mortality experience, estimated from both net and crude survival methods, and grouped in four age groups: 0-49, 50-59, 60-69 and +70. In some cases, the third and fourth age groups were merged due to the small amount of data. In other cases, even after merging these two groups results were still not meaningful (cf. Figures 4 (b), 5 (b), and 6 (b) below).

As the assumption about the percentage of censored observations that have experienced an event of death is a very important assumption, with impact on the conclusions, the sensitivity of the results to the adjustment was tested in all methods for four different mortality scenarios: 0%, 40%, 80% and 100%. We finally concluded that the assumption of 80% mortality allowed for a compromise between reality (healthy lives to have higher life expectancy than impaired ones) and the need to be conservative, as the main goal is to price enhanced annuities.

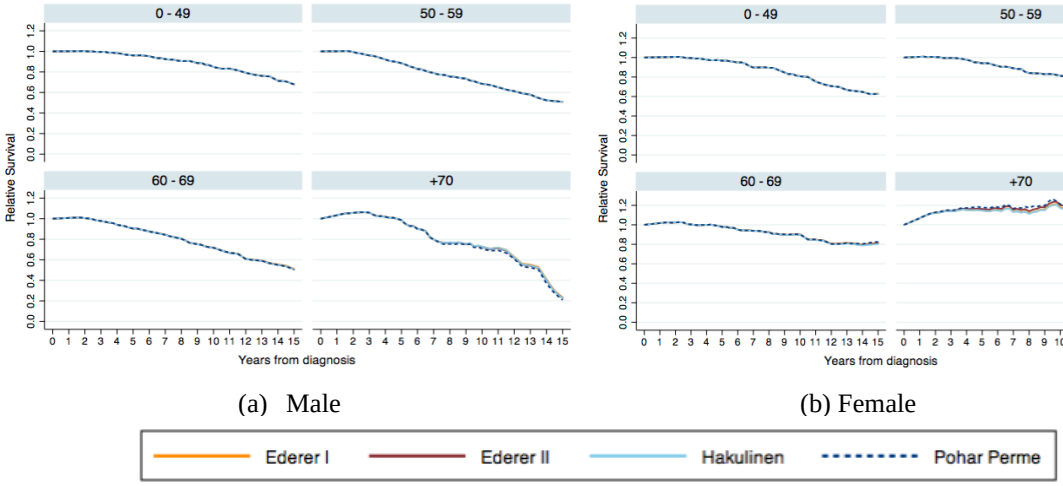
Net Survival Rates were estimated using Ederer I, Ederer II, Hakulinen and Pohar Perme, cf. equations (2)-(10) in the Appendix. Comparing the results, we can conclude that the four methods give very similar results, especially in the first ten years from the diagnosis.

4.1.1. NRS Cardiovascular diseases

Figures 3-4 show the relative mortality of two cardiovascular diseases, heart attack (Figure 3) and stroke (Figure 4). Graphs were also obtained for two medical conditions that are risk factors for cardiovascular diseases, hypertension and cholesterol, but since they are quite similar to Figures 3 and 4 there is no need to display them.

In general, the life expectancy reduction due to the diseases under study is greater for males than for females. For males, the negative impact of the disease is increasing with age group and years from diagnosis, which means that recoverability decreases with age. For females, excluding stroke disease, the relative mortality behaviour, from an age at diagnosis under 60, is very similar to the one for males. The relative survival is almost linearly decreasing with time from diagnosis, reaching after fifteen years from diagnosis the

values of, approximately, 60% for the first age group (0-49) and 40%-50% for the second group (50-59). This indicates that life expectancy for someone diagnosed with heart attack, hypertension or cholesterol before the age of 50 is lower than life expectancy for the reference population, approximated by life tables GKM95 and GKF95. Results in Section 4.3 confirm it.



(a) Male (b) Female

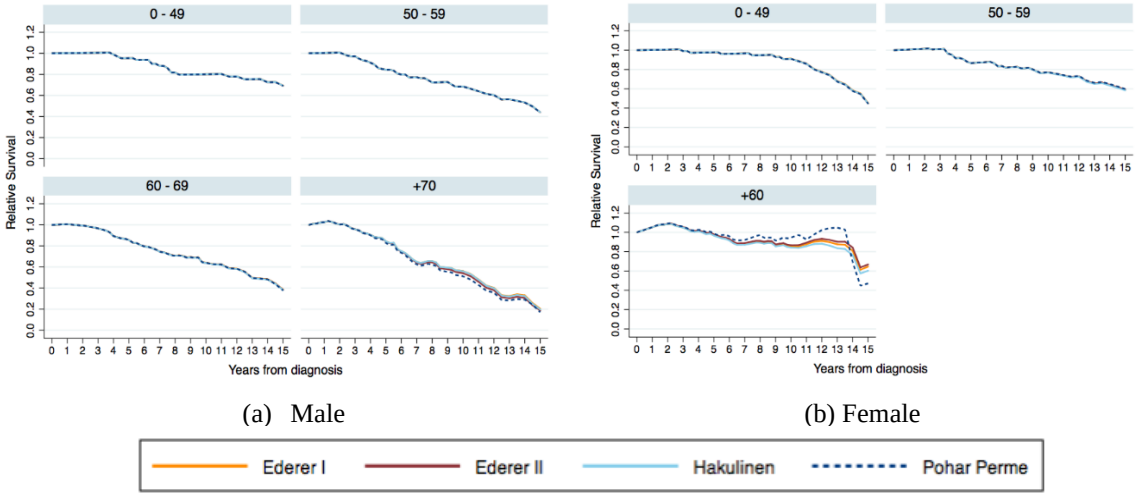
Figure 3. Heart attack – net relative survival by age in four categories, males and females. Source: Authors' calculations from SHARE database.

For some cardiovascular conditions, as heart attack, stroke and hypertension, the results obtained for the oldest female group, contrary to what would be expected, demonstrate a life expectancy greater for diseased individuals than for the healthy ones. These results are in broad disagreement with world statistics, showing that cardiovascular disease is the leading cause of death among global population, especially for elder women, which indicates the presence of bias in the estimates.

Relative survival models establish a comparison between life expectancy of the individuals diagnosed with the disease of interest and the reference population *free* of the disease. The assumption, previously discussed, that the presence of the disease under study in the general population only produces a very negligible effect on relative survival estimates, may not be fulfilled in this case. In fact, the same reason for the inconsistency is mentioned in Nelson *et al.* (2008), p.946, "*given the predominant contribution of heart disease to mortality in industrialized society, the appropriateness of this assumption in coronary heart disease needs to be assessed, in particular for oldest age groups*." Further reasons for the fact that the relative survival ratio at some

ages is higher than 1 (also observed in some of the figures below) are the significant number of missing deaths in the follow up process and the "health patient effect", see Bajpai, Chaturvedi and Pandey A (2014).

Although the number of people wishing to purchase annuities at such ages and conditions is quite small in Portugal, an acceptable solution would be to extrapolate the rates estimated to the previous age-groups, to guarantee that the price of hypothetical annuities purchased by people at this ages would not be higher than the prices of annuities purchased by healthy or younger people. By doing this, a compromise between the interest of the annuity provider and that of the annuitant would be achieved. In the numerical applications we follow this rule.



(a) Male (b) Female

Figure 4. Stroke – net relative survival by age in four categories, males and females. Source: Authors' calculations from SHARE database.

4.1.2. NRS Respiratory diseases

Two different respiratory diseases, chronic lung disease (Figure 5) and asthma (Figure 6), have been analyzed.

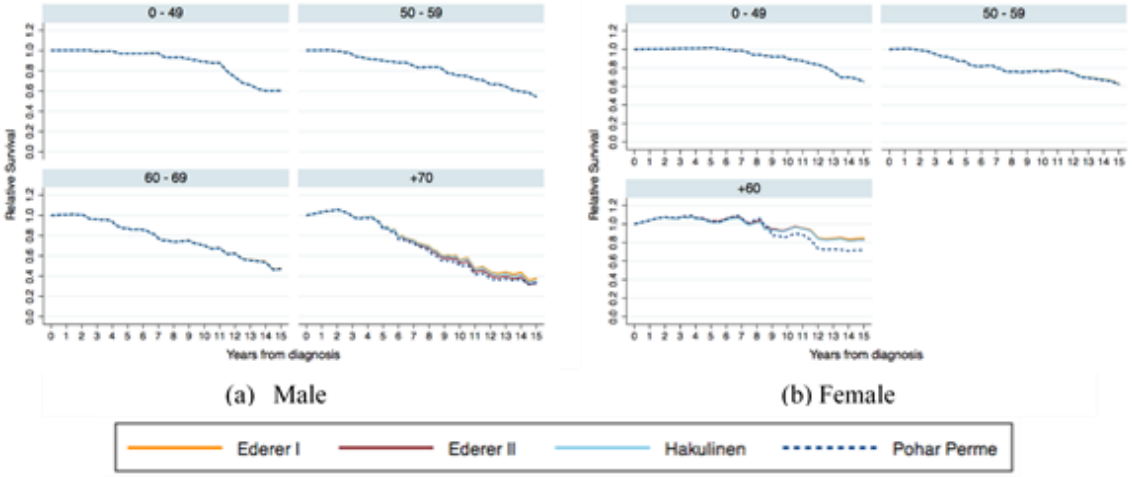


Figure 5. Chronic lung disease – net relative survival by age in four categories, males and females. Source: Authors' calculations from SHARE database.

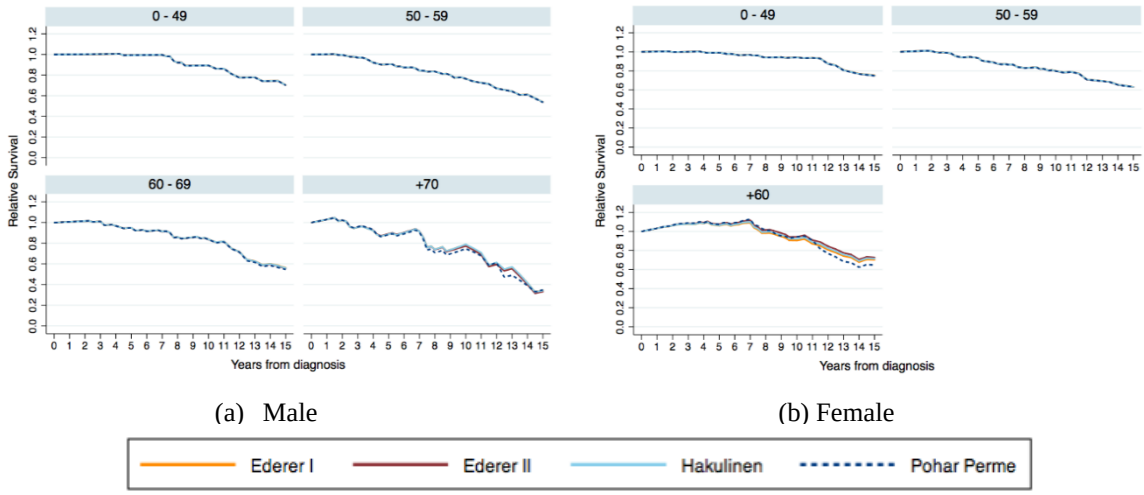


Figure 6. Asthma – net relative survival ratios by age in four categories, males and females. Source: Authors' calculations from SHARE database.

For both conditions, we can conclude that the negative effect on life expectancy seems to be substantially greater for males than for females. According to data from the Global Health Observatory of the World Health Organization (2019), standardized death rates for respiratory diseases are consistently higher for men than for women, which could be explained by different smoking habits and occupational risks. The negative effect on life expectancy is increasing with age at diagnosis and years from diagnosis. According to the same source, standardized death rates for diseases of the respiratory system were particularly high at advanced ages, explaining concerns over, for example, winter influenza epidemics.

4.1.3. NRS Cancer

Like the previous diseases, cancers figure among the leading causes of death. The Global Health Observatory of the World Health Organization (2019) predicts a rise of 70% in the number of new diagnosed cases during the following decades. Cancer is a generic term for a large group of diseases that could be responsible for different mortality patterns, depending on the type of cancer, the stage at diagnosis and the treatment followed. We will consider all types jointly since the dimension of the data does not allow for disaggregation. Figure 7 shows that relative survival rates decrease with the years from diagnosis. This effect is more pronounced for males than for females, indicating a possible higher recoverability for women.

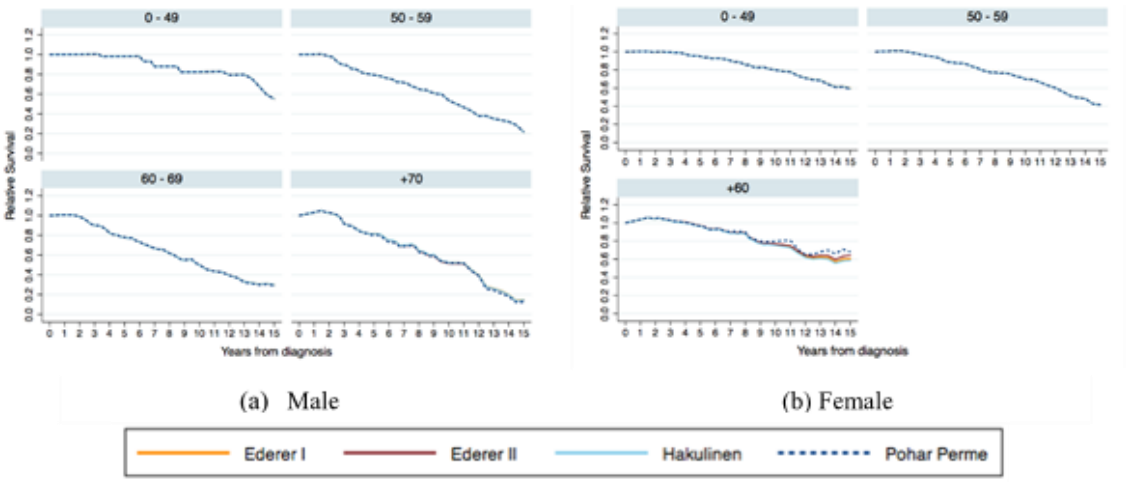


Figure 7. Cancer – net relative survival by age in four categories, males and females. Source: Authors' calculations from SHARE database.

For the male population, the negative impact of the disease on survival is, in general, increasing with age at diagnosis. The results show that the five-year probability of survival for someone under 50 diagnosed with cancer is almost the same as that for healthy individuals, and it is about 80% for higher ages at diagnosis. For the female population, there is no significant difference between the five-year survival of the healthy and the diseased population. The negative effect of the disease is less noticeable on older age groups. Again, one explanation is that the results are biased due to the fact that the reference population is not free of the disease, and cancer is a leading cause of death of the elderly women.

To conclude subsection 4.1, the crude relative death rates were computed by the Cronin and Feuer method, cf. equations (11) and (12) in the annex. Figures 8-10 show the outcome. To better distinguish the proportion of deaths explained by the disease of interest from the proportion explained by other diseases (competing risks), these figures show the cumulative probability of death.

4.1.4. CRS Cardiovascular diseases

For the group of cardiovascular diseases, the main conclusions are generic to both diseases, heart attack and stroke, and both medical conditions, hypertension and cholesterol. At younger ages, mortality in the group of individuals diagnosed with cardiovascular diseases is almost entirely explained by the disease of interest. Inversely, at the elderly ages the extra mortality of the diseased group over the healthy individuals is also likely to be explained by other diseases. So, the impact of other diseases on mortality experience is increasing with both age at diagnosis and years from diagnosis. Figure 8 illustrates the results for the heart attack case, the other three cases being quite similar.

The situation highlighted in comments to Figure 3 can also be observed here. The reference population is not free of the disease. In fact, cardiovascular problems, are the number one killer of elderly women. In Section 3.2 of their paper Cronin and Feuer (2000) discuss why "It is not unusual to observe relative survival greater than 1, which forces $\tilde{g}_{kc}$ to be less than zero - cf Equation (11) in the Appendix.

Cronin and Feuer (2000) further explain that it is possible to adjust results to eliminate the awkward probabilities, but this adjustment can lead to bias in

estimating cumulative probabilities of death. In this paper, results are kept as they were obtained, also to illustrate that this is not an unusual situation.

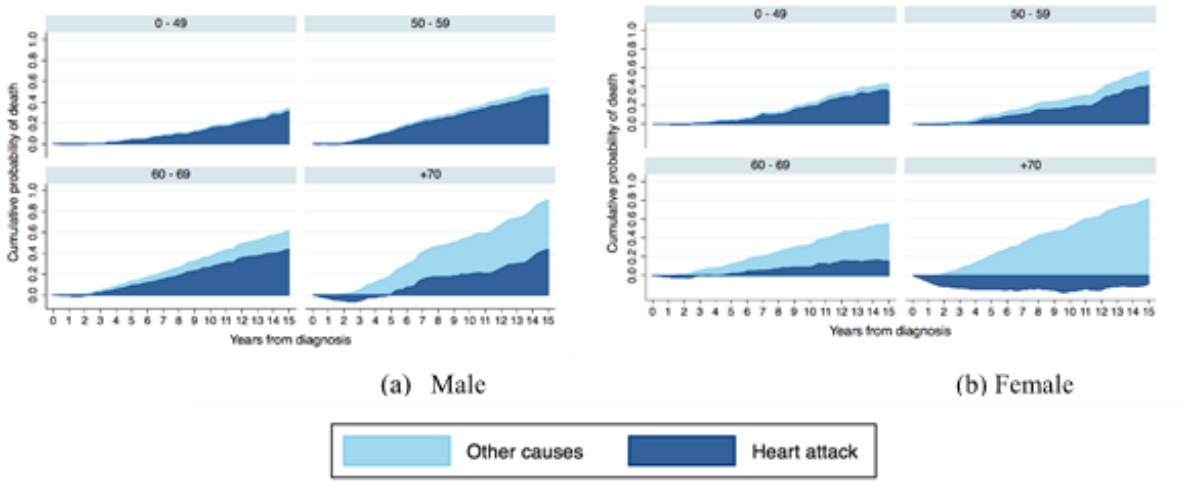


Figure 8. Heart attack – crude relative survival by age in four categories, males and females. Source: Authors' calculations from SHARE database.

4.1.5. CRS Respiratory diseases

For the group of respiratory diseases, chronic lung disease and asthma, the conclusions are similar to those of the cardiovascular disease group. The relative probability of death due to other causes is increasing with age at diagnosis of the disease of interest and years from diagnosis. In general, the effect of the diseases on mortality is greater for males than for females. Figure 9 illustrates for the chronic lung disease.

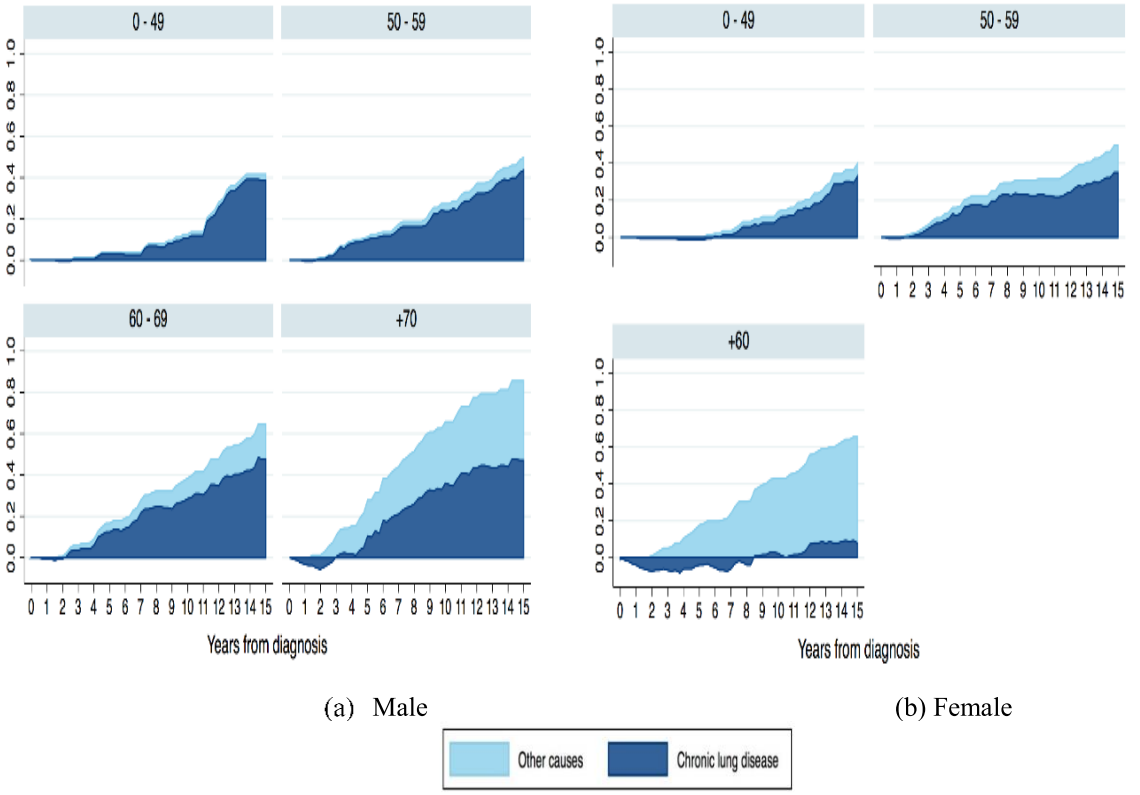


Figure 9. Chronic lung disease – crude relative survival by age in four categories, males and females. Source: Authors' calculations from SHARE database.

For the group of elderly women diagnosed with a respiratory disease, mortality due to the disease is lower compared with that of the reference population. However, the mortality rate due to other causes is substantially greater for this age group - the bias induced by the relevant presence of the disease in the reference population. Asthma increases the prevalence of concomitant diseases in the elderly population: "*It is known that asthma is associated with a specific pattern of comorbid conditions whose profile depends on age. Within the elderly population, asthmatics have an increased incidence of respiratory diseases, such as chronic bronchitis, chronic obstructive pulmonary disease and chronic sinusitis, but also stomach ulcers, cardiovascular disease, osteoporosis, diabetes, depression and cancer than the rest of the population.*" (Wardzynska, Kubsik and Kowalski (2015), p. 902).

4.1.6. CRS Cancer

Cancer results are in line with the previous ones (Figure 10).

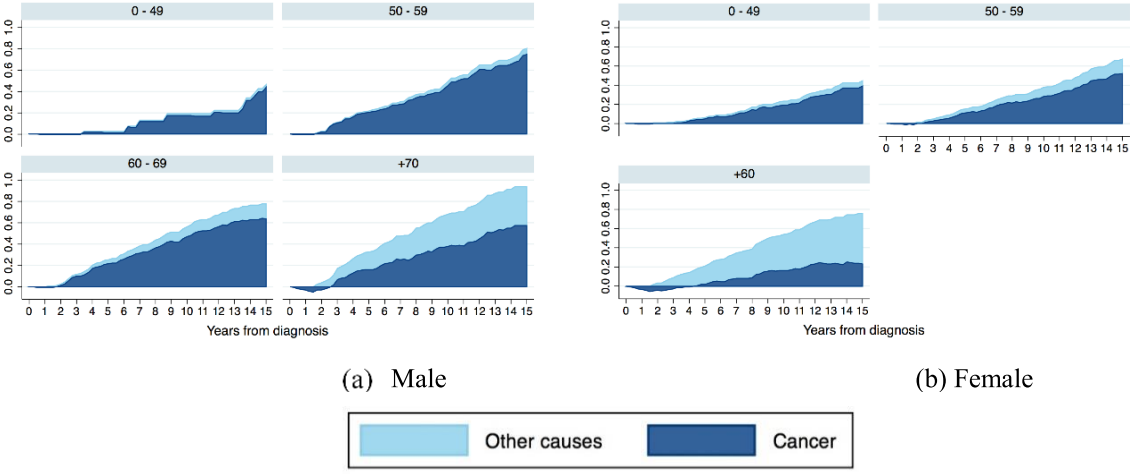


Figure 10. Cancer – crude relative survival by age in four categories, males and females. Source: Authors' calculations from SHARE database.

4.2. Impaired life annuities

4.2.1. Adjustments in the reference life table with net survival rates

Under the net measure and the relative framework, survival rates (and ratios) were estimated in the previous section using four different estimators. Since results are not significantly different, in this section the Pohar Perme's relative survival estimates, which the literature recognizes as less likely to be biased, are chosen. In this way, the number of diseased individuals alive at age x $\left({}^{net}l_x\right)$ is

$$ {}^{net}l_x = \begin{cases} l_{x'}, & x = x' \\ l_{x'} \times {}_{(x-x')}p_{x'} \times NS^{gr}_{(x-x')}, & x' < x \leq x'+15, \\ l_{x'} \times {}_{15}p_{x'} \times NS^{gr}_{15} \times {}_{(x-(x'+15))}p_{x'+15}, & x > x'+15 \end{cases} \qquad (13) $$

where: l_x is the number of individuals alive at the actual age *x*, from GKF95 and GKM95 life tables; x' is the age at diagnosis; ${}_k p_x$ is the probability of a life aged x' surviving to age $x'+k$, from GKF95 and GKM95 life tables; NS_k^{gr} is the net relative survival of the impaired lives in relation to the general population, by the Pohar Perme's estimator (10), *gr* being the age group at diagnosis. Four age groups were defined: $gr=1$ (below 50), $gr=2$ (between 50 and 59), $gr=3$ (between 60 and 69), and $gr=4$ (from 70 onwards). Here NS_k^{gr} can be interpreted as the relative probability that someone diagnosed with a certain disease at age group *gr* will survive *k* years after diagnosis, in relation to a healthy individual of the same age.

Equation (13), and also Equation (14), imply that a life who survives 15 years from diagnosis is considered to be cured. The recoverability clinical studies usually focus on the disease effect over mortality in a 5-year period. For the purpose of our study (enhanced annuities) the 5-year period is too short and this effect was estimated until 15 years from diagnosis. For simplicity, it was assumed the reference survival thereafter. Although some of the considered diseases have no cure, it seems to be a proper assumption since it is more conservative and less prone to influence the results. Since the ultimate purpose is to price life annuities, we may consider this assumption as a prudent assumption from the point of view of suppliers.

Figures 11 to 13 below permit a comparison of the survival function for the ages at diagnosis of 35, 55 and 75, between the reference population and the diseased group of individuals, both for males and females. The survival function for those who are impaired lives was computed using the number of individuals alive at age *x* estimated from equation (13) for each of all the diseases and medical conditions under analysis.

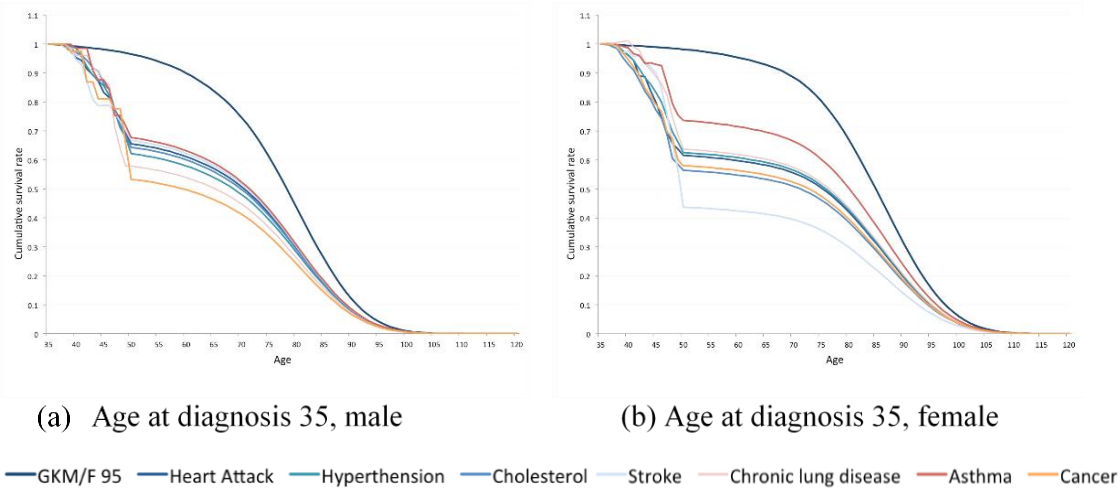


Figure 11. Net relative survival, age at diagnosis 35, males and females.
Source: Authors' calculations from SHARE database.

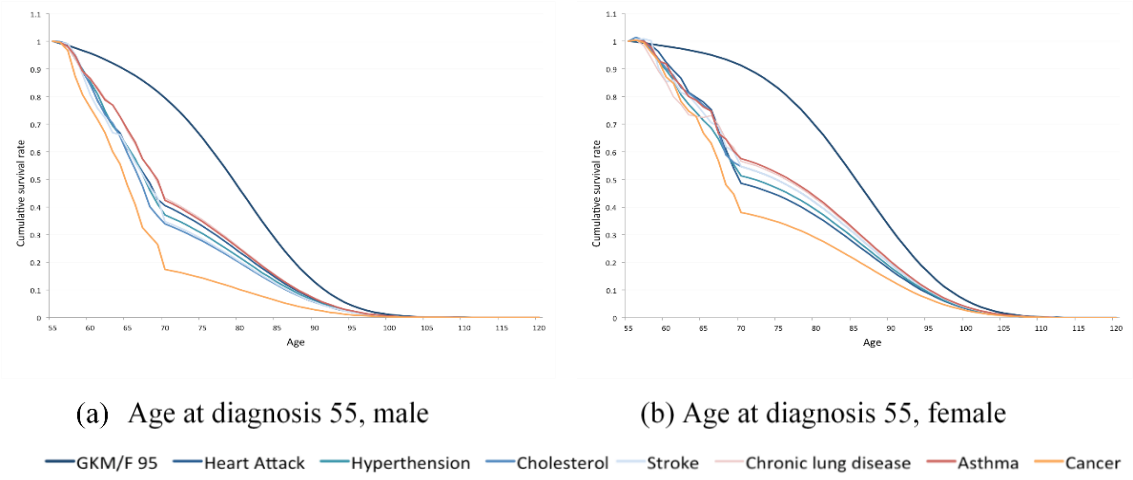


Figure 12. Net relative survival, age at diagnosis 55, males and females.
Source: Authors' calculations from SHARE database.

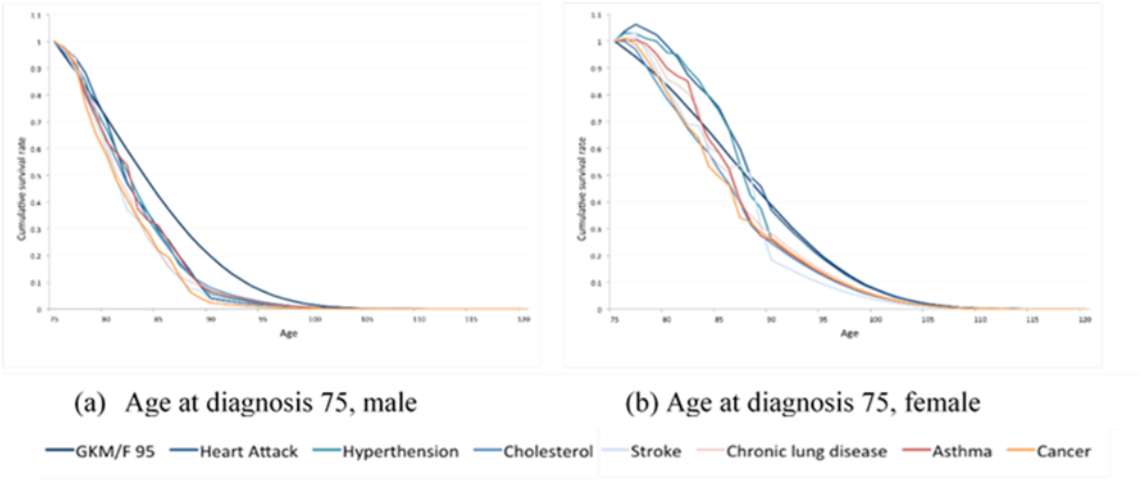


Figure 13. Net relative survival, age at diagnosis 75, males and females.
Source: Authors' calculations from SHARE database.

We can see that for an age at diagnosis of 55, for both male and female groups, the survival reduction is more pronounced for cancer, followed by cardiovascular and respiratory diseases. For the youngest age group, the impact of the diseases over the survival function strongly differ between genders.

For the male population cancer and chronic lung disease are in general the conditions with the greatest negative impact over survival, while for females the stroke disease and cancer are the ones with the most considerable impact. The elderly age group reveals that, for the male population, the presence of each disease tends to reduce the survival probability almost in the same proportion regardless of the disease in question. For the female population, as seen before, the most representative diseases in the reference population are responsible for the biased output observed.

4.2.2. Adjustments in the reference life table with crude probabilities of death

The results can be interpreted as the proportion of deaths in the diseased group in relation to the ones of the reference population, considering all possible causes of death beyond the disease of interest. The number of diseased individuals alive at age x is

$$
{}^{net}l_x = \begin{cases} l_{x'}, & x = x' \\ l_{x'} \times {}_{(x-x')}p_{x'} \times CS^{gr}_{(x-x')}, & x' < x \leq x'+15, \\ l_{x'} \times {}_{15}p_{x'} \times CS^{gr}_{(x-x')} \times {}_{(x-(x'+15))}p_{x'+15}, & x > x'+15 \end{cases} \qquad (14)
$$

where CS_k^{gr} is the crude relative survival of the impaired lives in relation to the general population, by the Cronin and Feuer estimator (11), computed for the four age groups at diagnosis (*gr*).

Figures 14 to 16 are analogous to the previous ones but, instead of the net survival, show the crude survival rates, calculated using the number of individuals alive at age *x* estimated from equation (14) for each disease.

Likewise previous results, at younger ages almost the entire mortality of the group of diseased individuals is explained by the disease of interest, and only for elderly age group the effect of the competing risks prevails. Thus, the survival probabilities computed with the two methods - crude and net survival estimates - are very similar for all ages, because at earlier ages the estimates are matching and at elderly ages the life expectancy is so low that corrections made to mortality become irrelevant.

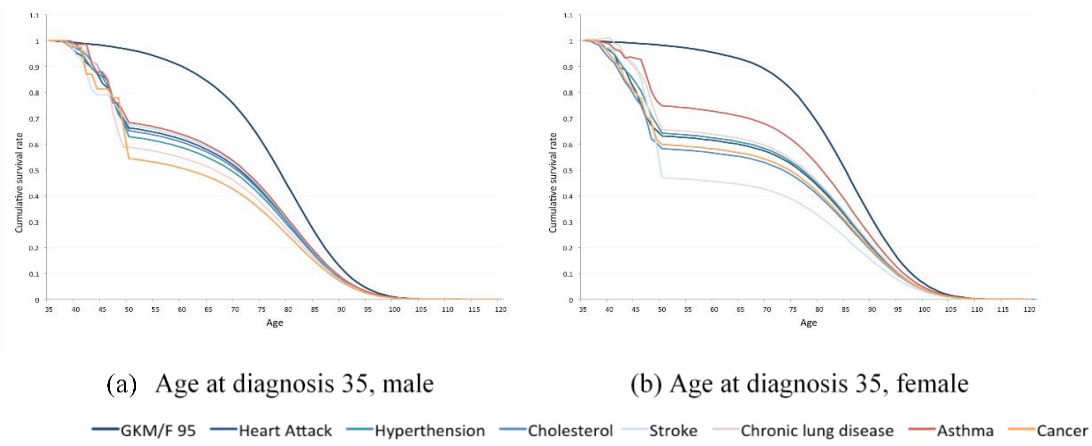


Figure 14. Crude relative survival, age at diagnosis 35, males and females. Source: Authors' calculations from SHARE database.

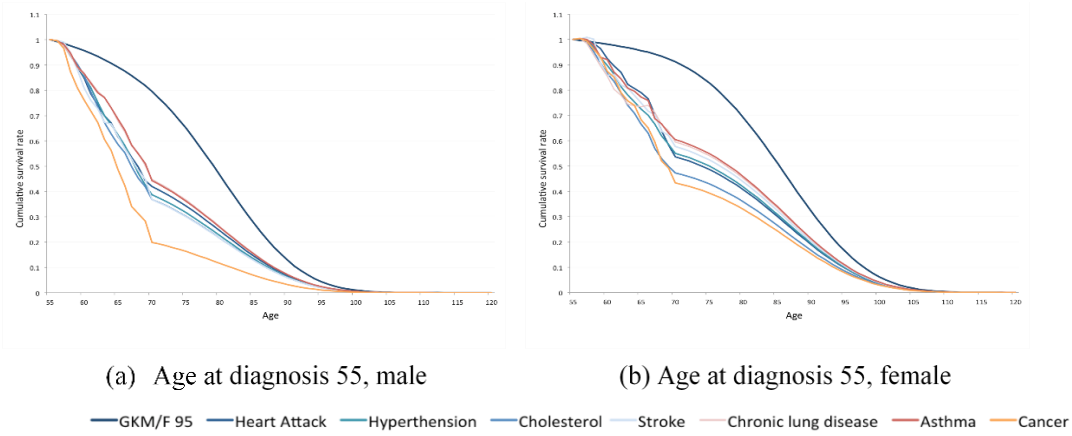


Figure 15. Crude relative survival, age at diagnosis 55, males and females. Source: Authors' calculations from SHARE database.

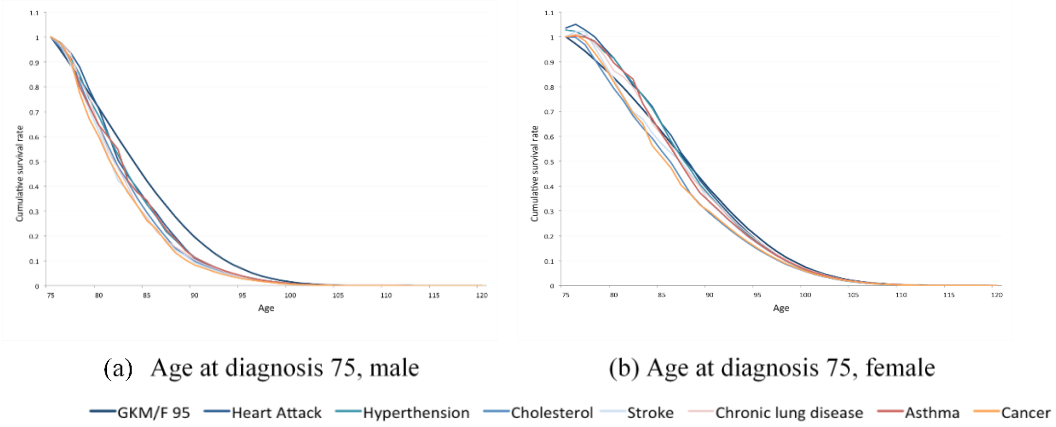


Figure 16. Crude relative survival, age at diagnosis 75, males and females. Source: Authors' calculations from SHARE database.

We conclude this section, by including Table 1 and Table 2, with the expectations of life for males and females, in years, for ages 40, 50, 60, 70, 80 and 90, and for the different diseases under study. They refer to the net survival measure, since the crude survival measure gives similar results, as seen in the previous paragraphs.

Table 1
Life expectancy in years, males (net survival)

	40	50	60	70	80	90
Heart attack	16.08	9.12	8.86	7.82	6.82	5.28
Hypertension	16.43	9.00	8.77	8.41	5.96	3.91
Cholesterol	17.73	9.21	8.43	8.05	5.36	4.28
Stroke	14.38	8.86	8.22	6.22	4.88	3.55
Chronic lung disease	15.99	10.06	8.45	7.08	5.46	3.88
Asthma	18.75	10.83	10.84	7.64	4.98	3.83
Cancer	15.20	8.08	7.33	6.16	4.17	3.52

Source: Authors' calculations from SHARE database.

Table 2
Life expectancy in years, females (net survival)

	40	50	60	70	80	90
Heart attack	15.70	12.44	9.38	6.07	3.79	1.72
Hypertension	17.12	12.44	8.75	6.17	4.46	1.20
Cholesterol	13.09	9.72	7.53	4.27	3.04	1.51
Stroke	16.82	11.32	8.68	6.28	4.37	2.68
Chronic lung disease	22.16	9.81	11.12	6.90	4.88	2.50
Asthma	23.06	12.94	12.32	8.06	4.49	2.63
Cancer	14.97	10.69	8.77	4.99	3.74	2.12

Source: Authors' calculations from SHARE database.

4.3. Life annuity premiums

The survival curves obtained will now be applied to calculate net premiums of immediate life annuities for impaired lives in Portugal. Recall (see Figure 2) that European data is used as a proxy for the Portuguese diseased population and tables GKM95 and GKF95 are used as a proxy for the general population. In order to compare the results, the annuities for both the reference and the diseased population (net and crude survival estimates) are computed considering the same main assumptions:

- a fixed annuity with twelve monthly payments ($m = 12$ below) of 1 unit of capital;
- an interest rate i of 2% - remark that the annuity premium values should be considered in relative terms rather than in absolute ones and small changes in the interest rate will very likely affect the numerator

and the denominator to similar extents. To analyze the sensibility of the ratio to larger changes of the interest rate in the different cases is beyond the aim of this paper and is a task for future work.

- the Uniform Distribution of Deaths.
- annuities are immediately purchased at the time of diagnosis.
- mortality by heart attack in the oldest female group is extrapolated from the rates estimated to other age-groups (due to inconsistent results).
- a life that survives 15 years from diagnosis is considered to be cured, as discussed in the paragraph following Equation (13).

Net life annuity premiums are calculated in the usual way, as the Expected Present Value of the payments made by the whole life annuity:

$$P = \frac{1}{m} \sum_{k \in \left\{0, \frac{1}{m}, \frac{2}{m}, \ldots\right\}} v_i^k \, {}_k p_x, \quad v_i^k = \frac{1}{(1+i)^k}, \tag{15}$$

where v_i^k is the discount factor and the probabilities ${}_k p_x$ are calculated using the adjusted tables, to reflect the lower survival of the impaired lives, and the Uniform Distributions of Death assumption, when *k* is not an integer.

In Tables 3 to 6, we observe that the annuity values computed with the reference life tables and those computed to impaired lives tend to be strictly decreasing with age, reproducing the life tables' age effect. A few exceptions to the general pattern are e.g. Heart Attack and Hypertension (males) where values are not strictly decreasing between age groups. This is explained by the fact that mortality adjustments have been computed by age groups. When the correction factors are applied to the reference life table, for each age at diagnosis, the reduction in life expectancy in the first 15 years is so significant that almost nullifies the life table's age effect for each age group. With only a few exceptions, results show that net and crude survival estimates do not produce significantly different annuity values.

For the group of cardiovascular diseases, the annuity values for the first age group are about less than a half of the reference annuity values. As expected, the annuity values for the second age group are lower than those for the first one, and the absolute differences between the reference and diseased populations tend to decrease as age at the beginning gets closer to the end of the life table.

Comparing the survival rates and the resulting life annuity premiums across diseases shows that cancer is the condition that most decreases life expectancy (and consequently the premiums), for both men and women from the age of

40. Other very serious conditions are stroke, cholesterol and heart attack for men and heart attack, hypertension and cholesterol for women, see Figures 11, 12, 14, and 15 and Tables 3-6 below.

Comparing life annuity premiums for men and women, mostly because mortality in table GKF95 is expressively lighter than mortality in table GKM95, both the absolute and relative differences of the value of annuities for female impaired lives are higher than those for men. As age increases, all the absolute differences decrease, as it should be, but the relative differences seem to stabilize for female lives. There are inspiring differences between the genders, but European legislation for anti-discrimination purposes is a barrier to implement any measure with this purpose in view. For interesting discussions on the topic, see Avraham (2017) and Rego (2015).

Table 3
Life annuity premiums, males (net survival rates)

	40	50	60	70	80	90
GKM95	306.9	249.5	187.8	126.8	78.8	48.4
Heart attack	133.3	81.6	85.3	89.8	69.6	48.7
Hypertension	136.2	83.1	84.4	82.9	64.8	45.8
Cholesterol	147.0	82.4	81.1	72.9	58.3	42.5
Stroke	119.2	79.3	73.8	64.0	53.1	40.2
Chronic lung disease	134.2	90.0	79.9	72.8	59.3	43.9
Asthma	155.4	91.1	101.5	75.5	59.9	43.3
Cancer	126.0	65.4	62.7	62.8	52.2	39.8

Source: Authors' calculations from SHARE database.

Table 4
Life annuity premiums, males (crude survival rates)

	40	50	60	70	80	90
GKM95	306.9	249.5	187.8	126.8	78.8	48.4
Heart attack	134.5	82.4	87.2	97.4	72.8	49.6
Hypertension	137.5	83.8	86.2	88.6	67.2	46.6
Cholesterol	148.4	81.8	84.0	74.7	59.2	42.8
Stroke	120.3	80.0	75.0	69.0	55.9	41.5
Chronic lung disease	135.6	90.9	81.4	78.8	62.5	45.1
Asthma	156.8	92.2	104.8	81.2	62.5	44.2
Cancer	127.1	65.8	63.5	66.8	54.6	40.9

Source: Authors' calculations from SHARE database.

Table 5
Life annuity premiums, females (net survival rates)

	40	50	60	70	80	90
GKF95	344.6	292.1	231.6	164.9	107	66.8
Heart attack	120.9	109	83.6	59.5	38.6	24.1
Hypertension	130.1	94.5	81.7	58.2	37.7	23.6
Cholesterol	105.6	103.7	76.1	54.2	35.2	22.0
Stroke	134.5	98.4	84.7	60.3	39.1	24.4
Chronic lung disease	169.6	86.2	93.0	60.3	39.1	24.4
Asthma	185.2	104.6	105.4	75.1	48.7	30.4
Cancer	114.3	87.3	73.3	52.2	33.9	21.2

Source: Authors' calculations from SHARE database

Table 6
Life annuity premiums, females (crude survival rates)

	40	50	60	70	80	90
GKF95	344.6	292.1	231.6	164.9	107	66.8
Heart attack	123.4	113.2	86.1	61.3	39.8	24.8
Hypertension	133	97.3	83.8	59.6	38.7	24.2
Cholesterol	107.5	82.6	69.1	49.2	31.9	19.9
Stroke	138.4	101	87.1	62.0	40.2	25.1
Chronic lung disease	174.2	88.7	95.6	68.1	44.2	27.6
Asthma	188.7	108.1	108.0	76.9	49.9	31.1
Cancer	116.5	89.7	75.0	53.4	34.7	21.6

Source: Authors' calculations from SHARE database

5. Conclusion

The life annuities market is underdeveloped in Portugal and passive adverse selection may be one of the causes. In this paper, we offer a contribution evidencing that is possible to price annuities for lives weakened by disease in a fair way. Using data provided by SHARE to calculate net and crude survival rates, we assess the impact of some serious diseases over a reference survival curve and then value life annuities for lives impaired due to these diseases.

The main findings suggest that, excluding some particular age and sex groups where the disease under study is strongly present in the reference population, the relative survival of the diseased individuals tends to be below the

reference, the gap increasing with age at diagnosis and years from diagnosis. The crude estimates achieved for the first age groups are very similar to net estimates. The differences between the two methods are only significant to the elderly individuals, inducing that the mortality of the younger individuals is almost entirely explained by the diagnosed disease while for the older ones different health problems tend to act simultaneously.

The values of annuities are expressively lower for impaired lives. This effect is particularly pronounced at younger ages and tends to disappear as age at the beginning gets closer to the end of the life table. Even assuming the cure of the diseased individuals 15 years after the diagnosis, which means we assume the reference mortality after that, the annuity premiums are substantially lower than the reference.

With this contribution, an attempt was made to suggest a direct way to estimate life annuity premiums for some groups of individuals with different life expectancies. However, we are aware that there is statistically significant information prone to influence the survival of these groups that is excluded. For future research, we would like to extend the study by including relevant information like the stage and the number of occurrences (relapses) of the disease, as well as personal information beyond the gender.

Nowadays, and for term life insurance products, this information is beginning to be used by insurance companies to exclude possible contracts or to apply discounts and extra premiums. Although there is legislation limiting underwriting factors, for social fairness and anti-discrimination purposes, the extra information in the case of pricing life annuities for impaired lives would work on the best interest of the annuitant. The use of this information in life annuity products would considerably widen the target scope and improve the fairness of the market.

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Appendix

In the following, let p_{ik-1}^{s} be the annual conditional probability that the life in the standard population (*s*) corresponding to the *i*-th life in the diseased population will survive interval $[k-1,k[, k=1,...,t.$ The two lives are similar with respect to age and calendar year of entry into the study and this probability is obtained from the standard life tables. The expected probability of individual *i* surviving until the end of interval $[t-1,t[$ is $S_i^s(t)=\prod_{k=1}^{t} p_{ik-1}^{s}.$ Formulas (2) to (10) next refer to the estimation of net survival probabilities and formulas (11) and (12) of crude death probabilities.

Ederer I - Exact Method

According to Ederer, Axtell and Cutler (1961), the cumulative expected survival probability from the beginning of the study to the end of interval $[t-1,t[$ is estimated by

$$\tilde{S}(t)^{EI}=\frac{1}{l_1}\sum_{i=1}^{l_1}\prod_{k=1}^{t} p_{ik-1}^{s}, \qquad (2)$$

where l_1 is the number of individuals at risk at the start of the first interval. Therefore, the estimate by the exact method is an average of the cumulative expected survival probabilities of all individuals in the diseased population.

Ederer II - Conditional Expected Survival Method

Ederer II Method (Ederer and Heise (1959)) requires two steps. The first step calculates the annual survival probabilities for each interval $[k-1,k[,\ p_{*k-1}^{s}=\frac{1}{l_k}\sum_{i=1}^{l_k} p_{ik-1}^{s},\ k=1,2,...t,$ where l_k is the number of individuals at risk (alive and not censored) at the start of the interval. The second step obtains the estimate of the survival function at time *t*,

$$\tilde{S}(t)^{EII}=\prod_{k=1}^{t}\left(\frac{1}{l_k}\sum_{i=1}^{l_k} p_{ik-1}^{s}\right)=\prod_{k=1}^{t} p_{*k-1}^{s}. \qquad (3)$$

Hakulinen - The Cohort Method

The cohort method for the estimation of the expected survival function was initially proposed by Hakulinen (1982). After specifying the potential follow-

up times for all individuals, define interval $\left[k-1,k\right[$. From the group of individuals with potential follow-up greater or equal to $k-1$ define two subgroups, the group α_k of the lives having a potential follow-up greater or equal to *k* and the group β_k of those having a potential follow-up less than *k*, i.e., individuals who withdraw during the interval. An expected life table can be constructed considering:

- the expected number of lives alive and under observation at time $k-1$,

$$l_k^* = \begin{cases} l_1, & k=1 \\ \sum_{i\in\alpha_{k-1}} S_i^S\left(k-1\right), & k\geq 2 \end{cases}; \qquad (4)$$

- the expected number of deaths among the α_k group during interval $\left[k-1,k\right[$,

$$d_k^* = \begin{cases} \sum_{i\in\alpha_k}\left(1-p_{ik}^s\right), & k=1 \\ \sum_{i\in\alpha_k} S_i^S\left(k-1\right)\left(1-p_{ik}^s\right), & k\geq 2 \end{cases}; \qquad (5)$$

- the expected number of lives withdrawing alive during interval $\left[k-1,k\right[$, by assumption (Hakulinen (1982)),

$$w_k^* = \begin{cases} \sum_{i\in\beta_k}\sqrt{p_{ik}^s}, & k=1 \\ \sum_{i\in\beta_k} S_i^S\left(k-1\right)\sqrt{p_{ik}^s}, & k\geq 2 \end{cases}; \qquad (6)$$

- the expected number of deaths among the β_k group during interval $\left[k-1,k\right[$,

$$\sigma_k^* = \begin{cases} \sum_{i\in\beta_k}\left(1-\sqrt{p_{ik}^s}\right), & k=1 \\ \sum_{i\in\beta_k} S_i^S\left(k-1\right)\left(1-\sqrt{p_{ik}^s}\right), & k\geq 2 \end{cases}; \qquad (7)$$

- the total expected number of deaths during interval $\left[k-1,k\right[$,

$$D_k^* = d_k^* + \sigma_k^*. \qquad (8)$$

The expected interval-specific survival probability is then estimated using the life table approach $p_*^S\left(k\right) = 1 - \dfrac{D_k^*}{l_k^* - w_k^*/2}$ and the procedure concludes by

obtaining the expected cumulative survival from the beginning of the first interval to the end of interval $\left[t-1,t\right[$,

$$\tilde{S}(t)^{H} = \prod\nolimits_{k=1}^{t} p_{*}^{s}(k). \qquad (9)$$

Pohar Perme Method

Perme, Stare and Estève (2012) discussed that the previous three estimators do not provide information on the mortality caused by the disease of interest that is independent of the national general population mortality, which means that they are not suitable for comparisons between countries.

To overcome the problem, a new estimator of net survival probability that enables the desired comparability between countries is recommended. It is an estimator for the continuous time assessment of the net survival, by weighting with the inverse of the individual-specific expected survival probabilities. The purpose of the weights is to inflate the observed person-time and number of deaths to account for person-time and deaths not observed as a result of mortality due to competing causes.

Dickman and Coviello (2015) provided a discrete version of net survival following the Pohar Perme approach, where weights are based on the cumulative expected survival at the midpoint of interval $\left[k-1,k\right[$. as follows

$$NS_k = \frac{1-\dfrac{d_k^w}{n_k^w - c_k^w/2}}{\exp\left\{-\dfrac{\sum_{j=1}^{n_k^w}\lambda_j^{*w} - \sum_{j=1}^{c_k^w}\lambda_j^{*w}/2 - \sum_{j=1}^{d_k^w}\lambda_j^{*w}/2}{n_k^w - \left(d_k^w + c_k^w\right)/2}\right\}}, \qquad (10)$$

where d_k^w is the weighted number of deaths during the interval, n_k^w is the weighted number of individuals alive at the start of the interval, c_k^w is the weighted number of censorings during the interval, and λ^{*w} is the weighted expected hazard, cf. Dickman and Coviello (2015), p. 191, for further details.

Cronin and Feuer Method

Cronin and Feuer (2000) developed a measure for the cumulative *crude* cause-specific probability of death, using relative survival instead of cause of death information. This is a method analogous to relative survival that conveniently measures mortality in the presence of other causes without the use of cause of death information.

The authors proposed to estimate the crude death rates, separately, due to the disease under study $\left(\tilde{g}_{kc}\right)$ and due to other causes $\left(\tilde{g}_{ko}\right)$, in the following way:

$$\tilde{g}_{kc} = \left(\prod_{r=1}^{k-1} \hat{P}_r\right)\left(1-\frac{\hat{P}_k}{E_k}\right)\left(1-\frac{1}{2}\left(1-E_k\right)\right); \qquad (11)$$

$$\tilde{g}_{ko} = \left(\prod_{r=1}^{k-1} \hat{P}_r\right)\left(1-E_k\right)\left(1-\frac{1}{2}\left(1-\frac{\hat{P}_k}{E_k}\right)\right), \qquad (12)$$

where: $\hat{P}_r = 1-d_r/n_r^*$ is the maximum likelihood estimator of the probability of surviving interval $\left[r-1, r\right[$, conditioned on surviving to the beginning of the interval, estimated using a life table approach; d_r is the number of people who died in interval $\left[r-1, r\right[$; $n_r^* = n_r - l_r/2$ is the number of people at risk during the interval, adjusted for uniform loss to follow-up, n_r being the number of people alive at the beginning of interval $\left[r-1, r\right[$ and l_r the number of people lost to follow-up in the interval; E_k is the expected net survival for other causes in interval $\left[k-1, k\right[$ conditioned on being alive at the beginning of interval (that is, the survival that the cohort would have expected if they did not have the disease under study); $\hat{P}_k / E_k$ is an estimate of net survival for the disease under study.

The cumulative estimates are, respectively, $\tilde{G}_{tc} = \sum_{k=1}^{t} \tilde{g}_{kc}$ and $\tilde{G}_{oc} = \sum_{k=1}^{t} \tilde{g}_{oc}$. The estimates proposed assume independent competing causes of death. For simplicity, the concept of latent time of death for each competing cause acting within a population is adopted. Under this concept, the probability of dying in interval $\left[k-1, k\right[$, conditioned on surviving to the beginning of the interval, can be written as $1-\left(1-h_{kc}\right)\left(1-h_{ko}\right) = h_{kc} + h_{ko} - h_{kc}h_{ko}$, where $h_{kc} = 1-\hat{P}_k / E_k$ and

$h_{ko} = 1 - E_k$ are respectively the probabilities that the latent times of death for the disease under study, and for other causes, occur in interval $[k-1, k[$.

Remark that Cronin and Feuer (2000) assume the uniform distribution for the time of death from the disease under study and the other causes, which implies that the probability of dying from the disease is $h_{kc}\left(1-0.5h_{ko}\right)$ and the probability of dying from other causes is $h_{ko}\left(1-0.5h_{kc}\right)$, allowing thus to define $\tilde{g}_{kc}$ and $\tilde{g}_{ko}$ as in (11) and (12).