

EVALUATION AND SCORING PROPOSAL FOR INTERNATIONAL PENSION SYSTEMS

EVALUACIÓN Y PROPUESTA DE SCORING SOBRE LOS SISTEMAS DE PENSIONES A NIVEL INTERNACIONAL

Diego Rodolfo Gorostieta Ortega

Facultad de Economía. Universidad Autónoma del Estado de México. Toluca de Lerdo, México.

ORCID: <https://orcid.org/0009-0003-3423-9773>

(Corresponding author)

Fecha de recepción: 9 junio 2025

Fecha de aceptación: 24 septiembre 2025

ABSTRACT

Pension systems around the world have undergone significant transformations in recent decades in response to profound demographic changes that affect their performance. Population aging, increased life expectancy, and declining birth rates have challenged the full development of these systems, despite various parametric and structural reforms. In this context, the present article proposes a scoring system to evaluate the performance of pension systems across different countries, taking into account the diversity of existing schemes. The evaluation is structured around three fundamental pillars: coverage, adequacy, and financial sufficiency, which aim to reflect the main functions a pension system should fulfil. To construct this measure, statistical projections based on time series are used, along with principal component analysis (PCA) techniques, which allow for the synthesis of multiple variables into a single indicator. As result, this approach facilitates cross-country comparisons, annual trend analysis, and the identification of areas for improvement in the design of more effective public policies.

Keywords: pension systems, demography, scoring, time series, principal component analysis.

RESUMEN

Los sistemas de pensiones en el mundo han enfrentado importantes transformaciones en las últimas décadas como respuesta a los profundos cambios demográficos que afectan su desempeño. El envejecimiento poblacional, el aumento en la esperanza de vida y la disminución de las tasas de natalidad han dificultado el pleno desarrollo de estos sistemas a pesar de las distintas reformas tanto paramétricas como estructurales. En este contexto, el presente artículo propone un scoring que permite evaluar el desempeño de los sistemas de pensiones en distintos países, tomando en cuenta los distintos esquemas existentes. La evaluación se estructura en torno a tres ejes fundamentales: cobertura, adecuación y suficiencia financiera, los cuales pretenden reflejar las principales funciones que debe cumplir un sistema de pensiones. Para la construcción de esta medida se utilizan proyecciones estadísticas basadas en series de tiempo, junto con técnicas de análisis de componentes principales (PCA), que permiten sintetizar múltiples variables en un único indicador. Como resultado, este enfoque promueve la comparación entre países, el análisis de tendencias anuales y la identificación de áreas por mejorar dentro del diseño de políticas públicas más eficaces.



Palabras clave: sistemas de pensiones, demografía, scoring, series de tiempo, análisis de componentes principales.

1. INTRODUCTION

Nowadays, societies around the world are facing big demographic transitions, some more pronounced than others, but since the formation of the greatest ancient civilization, this is the first time that humanity is experiencing an unprecedented event, unique in history, which is happening as result of the combination of two phenomena: the increase in life expectancy and the decrease in fertility rates.

Worldwide, 2020 was a memorable year, the population aged 60 or over was greater than the population under five years old. In this way, the problem of population growth, which until a few decades ago seemed to focus on quantity, has been extended to an intergenerational imbalance in terms of older people. It is expected that in 2050 the proportion of the population over 60 years will double globally and reach more than 20% of the total population (World Health Organization, 2022).

Furthermore, population growth is expected to increase at a controlled rate in America, Europe, and Oceania, while in Africa and Asia, projections for the upcoming years indicate exponential increases. On the other hand, worldwide life expectancy at birth over the next thirty years is estimated to increase by approximately five years because of medical and technological innovations. In this context, it is essential to pay attention to the economic subsistence of people during old age. Firstly, the increase in life expectancy implies that people depend for longer on the money earned throughout working life and destined for old age. Undoubtedly, the problem is not limited to presenting the consequences of greater adversity to people with middle and low income (it is predicted that by 2050, almost 80% of the population will be concentrated within these incomes brackets) because subsistence conditions will lead to rise in migration rates, where the regions of Europe, North America and Oceania will be the main recipients of migrants from Africa, Asia and Latin America (United Nations, 2017).

In the 1880s, in Germany, Chancellor Otto Von Bismarck created a social security system, defined as a contributory pension scheme for active workers in relation to sickness and retirement benefits (Scheubel, 2013). Decades later, in England, William Henry Beveridge introduced a welfare plan that guaranteed a minimum pension of equal amounts for all workers with the main objective of combating poverty and assuring subsistence in the so-called "Beveridge Report" of 1942 (Conde-Ruiz & González, 2018)¹.

¹ Concerning the history of pension systems the Aerarium Militare was a financial institution that managed the payment and retirement of Roman pensioner soldiers (Fernández, 2003). Although this system declined along with the Western Roman Empire, during feudal times, pensions reemerged as non-formal systems based on the privileges of the nobility and the clergy, which, as an annuity, granted lifelong income from the commoners.

In the 16th century, with the consolidation of European states, the English Parliament granted annual pensions to the soldiers who lost limbs or became disabled in the service of The Crown. These pensions were financed by tax collection. Later, the British colonies of Plymouth and Virginia, in America, adopted this system for their militia. In the following years, with the arrival of the 18th century, British officials, motivated by the desire to obtain adequate income for those who could not continue in service, adopted a pension system in which a mandatory retirement policy stood out. At the same time, in France, the ministers who had led the French Revolution in 1789, granted themselves pensions under the argument that it was a compensation for their service, advocating for their autonomy (Clark *et al*, 2003).

On the other side of the world, pension systems in Latin America appeared for the first time in Chile, in 1924, under a defined benefit scheme. However, by 1980, Chile introduced a transformative establishment of a pension system, completely different from those known and which was called a defined contribution system (Subsecretaría de Previsión Social, 2024).

Today, most countries in Europe have a defined benefit system, because of adjustments and parametric reforms of the original systems of Bismarck and Beveridge. Although there are multiple classifications and methodologies to analyse various pension systems around the world, this study adopts an analysis based on cultural and regional areas that facilitates the understanding of the interrelations among the coexisting models, which are outlined below. In essence, pension systems in Europe are divided into the Anglo-Saxon model (United Kingdom), Central European model (Germany, France, and Poland), Southern European model (Spain and Italy), and Nordic Model (Sweden, Holland and Denmark) (Ablanedo & Baron, 2020). Whereas for Latin America, countries such as Argentina, Bolivia, Chile, Mexico, El Salvador, Peru, Colombia, Costa Rica, Uruguay, the Dominican Republic, and Panama, have adopted, after several structural and parametric reforms, defined contribution systems (Mesa-Lago, 2020). It should be noted that the constant parametric reforms in the pension systems are clear examples that reflect the dynamic needs of society. As a background to this problem, since the beginning of the century, Jiménez and Cuadros (2003) highlighted the fact that the deficiencies of the pension systems that existed at that time in Latin America, derived from the poor structural implementation of incentives, demographic changes, and low levels of the formal labour market.

Having described all these circumstances, we proceed to propose some important elements that must be considered when talking about pensions around the world and are closely related to the social organization of the countries where the different systems are implemented. These elements are immersed in the political structure, demographic and economic development, educational progress, and the awareness of citizens towards their financial well-being. In addition to that, the statistics available for each system, allow making comparative studies; although they are not approached with complete homogeneity due to the differences in each country, these offer an overview of social security. There is literature covering the evaluation of pension systems, where the approach is carried out from a conceptual point of view, studying the elements of the pension systems separately and most of the time focusing on the financial aspect as are Barr and Diamond (2006), Knoef *et al* (2016), Pokorný (2020) and Mesa-Lago (2022). Some interesting studies focus on the concept of adequacy such as Cole and Liebenberg (2008), Bigss and Springstead (2008), Guven and Holzmann (2009), Borella and Fornero (2009), Chybalski (2015) and Chybalski and Marcinkiewicz (2016). Particularly, Alonso-Fernández *et al* (2018) defines a synthetic indicator that measures the adequacy of the pension systems view since the income ratio and risk of poverty, benefit ratio, theoretical replacement rate, gender gap, old age dependency ratio and the employment rate.

In this sense, a new scoring system that considers how many people receive pensions, the pension amounts, and the financial sustainability of the pension system could offer a valuable perspective and a meaningful analysis of the efficiency of pension systems, approached from the fundamental social and financial elements of these systems. Additionally, a metric capable of integrating all these elements into a single data point would be of great interest in both academic and professional fields, as it would allow for general comparisons between countries while also enabling temporal comparisons that reflect the evolution of pension systems within each country. Ultimately, the goal is to identify areas for improvement for the benefit of the population.

2. OBJECTIVES

Given the contextualization of demographic conditions and the historical changes, both parametric and structural, made in pension systems around the world in the last years, it is important to review data about the development of pension systems, to build general overviews and subsequently,

encourage preparatory actions on the way to a challenging future. However, this information is the basis of a broader study, which seeks to maximize the availability and significance of the data.

This paper aims to carry out the analysis of different variables that are considered to be directly related to the performance of pension systems, to build an evaluation technique from them, with the objective of classify different countries. And in this way, getting a measurement that brings together the main aspects of a pension system (coverage, adequacy, and financial sufficiency), to achieve an annual evaluation of the conditions of the pension system of each country in a general glance, and after towards, showing international differences and promoting improvements in social protection.

The development of this analysis is approached in four sections. In Section 3 the methodology and the mathematical procedures addressed are described. In Section 4 the results are detailed including the findings from the application of the models, which are the basis of the discussions section (Section 5). Finally, in Section 6 are presented the general conclusions of the study.

3. METHODOLOGY

In recent years, payments in a pension system have become uncertain due to the great challenges faced by institutions. Talking about social security today, means stirring up controversy and harsh opinions on issues of public health, labour rights, and, particularly, pensions. Although the introduction of Personal Retirement Plans, mostly offered by insurance companies, has been a novelty in Latin America and some European countries, the economic and social reality of many countries has adopted them as products for a small population, even with the attractive benefits they offer. Consequently, economic stability in old age focuses on the efforts made during working life and the effectiveness of public programs.

The four most important components of pension systems, as defined by the Inter-American Conference on Social Security (cited in Contreras, 2020), are:

1. The amount of pension payment and the frequency of the payments.
2. The age at which the pension begins.
3. The period over which the pension will be paid.
4. The definition of actuarial equivalence.

However, within the elements that constitute pension systems, the principle of actuarial equivalence is essential for understanding the system. In summary, this actuarial approach dictates that there must always be equality between the amount of income and the amount of expenses, to guarantee full compliance with obligations.

According to Contreras (2020), regardless of the type of pension system in question, it must consider the following substantial elements to ensure good performance in most societies:

- 1- Monetary benefit.
- 2- Duration, whether for life or by regulations².
- 3- Frequency of payments.
- 4- Constant real value (ensure purchasing power).
- 5- Coverage for old age, death, and disability.

Furthermore, pension systems can be classified into different categories depending on the perspective from which the study is carried out: by type of scheme (transfers, insurance, and savings), by their target population (old age, disability, and survival), by type of administration

² In some countries, such as Chile, Peru, and Colombia, people can choose between programmed retirement plans or lifetime pensions, provided they meet the minimum requirements established by law.

(public or private), by its source of resources (direct or indirect) and by its financing system (defined benefit systems and defined contribution systems) (Contreras, 2020).

Bodie *et al* (1988) provide a description written that summarizes in a very understandable way what a defined benefit system and a defined contribution system consist of. In the first case, a defined benefit system focuses directly on the amount to be received, and it is calculated mathematically based on the years of service and the salary earned during the productive stage. At any point in time, the value of the pension can be seen as a deferred nominal annuity, because it cannot be received until retirement age, and the retirement amount is fixed.

On the other hand, in a defined contribution system, both employer and employee make contributions to a retirement account owned by the worker; these contributions can be invested in bonds, stocks, and investment funds previously authorized for this purpose. At the beginning of the retirement stage, the worker receives a sum because of the value accumulated in the fund which is regularly provided as an annuity during old age.

Now, as seen from the classification provided by Contreras (2020), defined benefit systems are financed through a pay-as-you-go model. In its pure form, a pay-as-you-go system distributes the costs represented by pension payments for a given year among the contributions of the economically active population. However, these pure models are not the most common. Overall, over 60% of the pension assets are managed by pension funds, while the remaining assets are managed by insurers companies and investment institutions (Social Security Administration, 2023). In summary, most of these defined benefit systems are state-managed and backed by the government to prevent defaults on current pension payments in case of funding shortfalls.

In contrast an individual capitalization model finances a defined contribution system. Under this scheme, although the amount of each contribution is determined, the benefit that will be received in the future is unknown. Normally, minimum benefits are guaranteed as benchmarks for contracting annuities with insurance companies, likewise, minimum pensions are usually funded through taxes when individuals lack sufficient contributory resources.

Once the elements and structure of both systems have been defined, we proceed to establish the axes that, since an own criterion are considered the foundations of the main areas of pension systems. These axes are coverage, adequacy and financial sufficiency. The coverage is the proportion of people of pension age who receive a pension. This was the simplest way that we found to deal with the accessibility of the pensions systems since a global sense, without consider gender, earnings, etc (Meneu-Gaya *et al*, 2020). Defining the concept of adequacy is a very difficult task, although an easy way to see if a pension system is adequate is thorough the prevention of poverty in old people and the maintenance of a reasonable living standard live in the retirement. In this case, we study adequacy under the future pensioner incomes first line, which is determined by the current replacement rate and can be updated for each subsequent year of analysis. However, it is important to recognize that there are different approaches since it is possible to study the adequacy (see e.g. Alonso-Fernández *et al*, 2018 and Rosado-Cebrian *et al*, 2020). The final aspect concerns to the financial sustainability. Comprehensive, the sustainability could be measure with the relation of pensions and contributions flows. With this we can determine the average proportion balance for each pensions system according to its characteristics (Devesa-Carpio *et al*, 2020). In this paper, we define sufficiency as the axis that encompasses the financial sustainability aspect. Punctually, we use the minimum expenditure basket to represent the expenses of the pension system, rather than the pension amounts, due to their variability and limited availability in databases.

In brief, these elements seek to ensure that the entire elderly population has sufficient income to survive in their retirement stage in the short, medium, and long term.

Mathematically we define:

Coverage:

$$Coverage = \frac{Pensioned\ population\ over\ retirement\ age}{Population\ over\ retirement\ age}. \quad (1)$$

As it is shown, a rate is constructed from population over the mandatory retirement age of each country pensioned and population over the mandatory retirement age of each country data. With (1) we can get the coverage as the proportion of the population over retirement age that have access to the pension scheme. For the countries of Austria, Belgium, Canada, the Czech Republic, Denmark, Finland, France, Hungary, Iceland, Japan, Portugal, and Slovenia, we found that the number of pensioners over the retirement age was greater than the total population over retirement age. However, this discrepancy is because the available data were collected from different sources. To address this, we converted these values of coverage to one. In the same way, the desired value is one, and in this case the coverage is total.

Adequacy:

$$Adequacy = \frac{\min (Average\ replacement\ rate, Replacement\ rate\ of\ 70\%)}{Replacement\ rate\ of\ 70\%}. \quad (2)$$

Replacement rate results from dividing the pension rights by the income in the productive stage. It indicates how good the provision of income in old age is, compared to the main provision before retirement without considering the application of taxes and contributions. Various experts on the subject have spoken regarding the replacement rate that is sufficient for the correct coverage of needs. Palmer (1989) states that a replacement rate of 60% to 75% is sufficient to enjoy an adequate standard of living; Yuh (2011) states that target replacement rates range from 65% to 85% depending on income level and marital status; likewise, the Presidential Advisory Commission on the Chilean Pension System for the year 2013, explained in a report on replacement rates, that the minimum sufficiency level is over 70% for the average income of the last years of the labour stage. For this study, a mandatory replacement rate of 70% is considered minimum and sufficient, additionally, to obtain values within an interval between 0 and 1, a maximum limit was established. From (2) is possible compare how much of the minimum replacement rate is reached by the replacement rate of each country. In other words, the adequacy indicates that, as greater is the value in (2), greater is the level of satisfaction of basic needs of the retired people. It is important to note that we use the gross replacement rate provided by the OECD.

Sufficiency:

For sufficiency, its approach becomes more complex compared to the two previous elements. It is important to highlight that different procedures are carried out depending on the type of system in each country, but for both cases the main idea is focused on get a measure that sum up the distribution of the incomes of the pension scheme, between the expenses of pensioned people. With this, although for each scheme the formula for sufficiency is different, we make it quantitatively comparable for each country.

In the case of countries with a defined benefit system, it is proposed to determine the value directly as a ratio of the incomes of all the working population that contributes to the system between all the consume of retired population receiving pensions. We define:

$$Sufficiency = \min \left(\frac{\sum_{j=1}^m Future\ economically\ active\ population_j \cdot Future\ contribution_j}{\sum_{j=1}^m Future\ pensioned\ population_j \cdot Future\ cost\ of\ minimum\ expenditure\ basket_j}, 1 \right). \quad (3)$$

In other matters, for the defined contribution systems, the aim is to obtain the value accumulated in the fund by a worker who begins his productive stage at the age of 22 and continues uninterruptedly until the effective age of exit from the labour market of each country. Thereafter, the expenditure that people will make during their retirement stage is approximated, according to retirement age and life expectancy. Mathematically, we have:

$$Sufficiency = \min \left(\frac{\sum_{i=1}^n (1+r)^{n-i} Contribution\ made_i}{\sum_{j=1}^m Future\ cost\ of\ minimum\ expenditure\ basket_j}, 1 \right). \quad (4)$$

In both cases:

$n :=$ years of contribution

$m :=$ years of expectancy life at retirement age

$r :=$ average rate of return of the investement of pension plans.

As it can see in (3) and (4) the principal idea is distributing the incomes between the expenses, while in (3) we made it in a collective manner, in (4) it is individually. It is worth highlighting that in both cases we propose the formulas attached to the conceptual ideas of each pension scheme. At the same time, it has maximum level of one with the objective of have all the axes of the methodology in the same units, in (1) to (4). Higher values indicates that the pension schemes can maintain itself financially over time. An uncommon case was Iceland, which was the unique country that due to the higher contribution rate and annual salary overcomes the maximum value of one in the sufficiency.

Implicitly, for the mathematical construction of sufficiency, in both cases, the following elements are required:

- 1) Average annual salary.
- 2) Contribution rate.
- 3) Normal retirement age.
- 4) Consumer price index.
- 5) Average annual rate of return on the investment of pension plans over the last 20 years.
- 6) Minimum expenditure basket.
- 7) Economically active population.
- 8) Pensioned population.

Specifically, to estimate future average annual salaries, the future economically active population, the future pensioned population and the future consumer price index, we use ARIMA models. Then, the numerator of equation (3) is determined using the projected economically active population, the projected average annual salary, and the contribution rate. The denominator, on the other hand, is calculated using the projected pensioned population and the future cost of the minimum expenditure basket. In contrast, for the numerator of equation (4), the projected average annual salary, the average rate of return and the contribution rate are used, while the denominator considers only the future cost of the minimum expenditure basket.

It is important to mention that for defining coverage, adequacy and sufficiency, most of the variables were obtained from the data provided by the Organization for Economic Cooperation and Development (OECD) on its OECD Data Explorer website³ except the minimum expenditure basket (which was obtained from the United States Bureau of Labor Statistics⁴) and the number of pensioners (obtained in the websites detailed in the Annex 1). The data of the OECD belong to the mandatory schemes of the Tier 2 (mandatory savings system, provided by either the public or private sector), however there are another classifications of the Averting the Old Age Crisis report of the

³ Source: <https://data-explorer.oecd.org/>

⁴ Source: <https://www.bls.gov/opub/reports/consumer-expenditures/2022/home.htm>

World Bank (World Bank Group, 1994), which includes Pilar 1 (a mandatory, publicly managed, tax-financed public pension), Pilar 2 (mandatory, privately managed, fully funded benefits) and Pilar 3 (voluntary, privately managed, fully funded personal savings) or the World Bank's Pension Conceptual Framework (World Bank Group, 2008) that contains Zero Pillar (a non-contributory basic pension from public finances to deal explicitly with the poverty-alleviation objective), First Pillar (A mandated public pension plan with contributions linked to earnings, with the objective of replacing some preretirement income), Second Pillar (Typically, mandated DC, with individual accounts in occupational or personal pension plans with financial assets), Third Pillar (Voluntary and fully funded occupational or personal pension plans with financial assets that can provide some flexibility when compared to mandatory schemes) and Fourth pillar (A voluntary system outside the pension system with access to a range of financial and nonfinancial assets and informal support, such as family, healthcare and housing).

All the data were searched in official websites and since the type of statistical information we assume that the number of pensioners of each country is an approximation of the number of pensioners that belongs to the mandatory schemes in the proposed model. Also, it should be noted that, for simplicity in the construction and evaluation of the model, the presented metrics were proposed knowing that they can be modified by incorporating more financial elements or taking a more complex concept. In the same sense, the definition of sufficiency adheres to the pure definition of each system: that is, government interventions to complement pension payments are not considered.

Of the OECD member countries, only those with sufficient data for the development of the model will be taken into account for study, but also those that maintain a constant pension system in the long term. These countries are reduced to 17, which are listed in Table 1. Based on the information collected: the type of system, retirement age, and average working lifetime, considering that it begins at age 22 for all countries and expected retirement time, which is constructed by subtracting the retirement age from the life expectancy (calculated at the time of retirement age).

COUNTRY	PENSION SCHEME	RETIREMENT AGE	AVERAGE YEARS OF WORK	LIFE EXPECTANCY	RETIREMENT EXPECTANCY	CONTRIBUTION RATE
AUSTRALIA	DC	67	45	86	19	
AUSTRIA	DB	65	43	84	19	
BELGIUM	DB	65	43	84	19	
CANADA	DB	65	43	85	20	
CZECH REPUBLIC	DB	64	42	82	18	
DENMARK	DC	67	45	84	17	
FINLAND	DB	65	43	84	19	
FRANCE	DB	64	42	87	23	
HUNGARY	DB	65	43	78	13	
ICELAND	DC	67	45	85	18	
JAPAN	DB	65	43	88	23	
PORTUGAL	DB	66	44	86	20	
SLOVENIA	DB	62	40	85	23	
SPAIN	DB	65	43	87	22	
SWITZERLAND	DB	65	43	87	22	
TÜRKİYE	DB	52	30	81	29	
UNITED STATES	DB	66	44	83	17	

Table 1: Description of some of the main elements of each country according to the pension scheme, where DC stands for Defined Contribution and DB stands for Defined benefit. The contribution rate was obtained from the OECD and is calculated based on the taxable income base. Source: author's own work.

4. RESULTS

4.1 Time series analysis for minimum expenditure basket, economically active population, and average salaries

Since we use annual data and aim to identify growth patterns for aggregate projections, ARIMA models (see Annex 2) are used to forecast the future economically active population, the future consumer price index, the future average salary and the future pensioned population. Once these are obtained, future contributions are calculated as a fixed rate of future salary and future minimum expenditure basket is obtained inflating the cost of the minimum expenditure basket for each country in 2022. It is important to emphasize that the coverage and adequacy calculations are obtained directly with the data provided by the OECD database; however, in the case of sufficiency, a more elaborate process is required, which begins with the collection of historical data and continues with the adjustment and development of models. Projections were made using ARIMA models, with forecasts beginning in 2023, the same year in which the pension schemes were evaluated using the proposed methodology⁵. The models covered the following variables: the consumer price index (used to estimate the cost of the minimum expenditure basket), the economically active population, and average salaries. All variables were modelled on an annual basis, resulting in the development of 51 models in R (R Development Core Team, 2024), specifically using the forecast package (Hyndman & Khandakar, 2008; Hyndman *et al*, 2024). Each ARIMA model was developed with all the annual data available, which were different for each country, with the “# Obs” column in Table 2 indicating the number of observations used for each one. Likewise, the residuals of the models were tested with the Shapiro-Wilk test for normality and the Ljun-Box test for autocorrelation, and for all the tests the p-value was greater than 5% (see Table 2 for models and p-values).

COUNTRY	CONSUMER PRICE INDEX			ACTIVE ECONOMIC POPULATION			AVERAGE SALARIES			PENSIONED POPULATION		
	SHAPIRO-WILK	LJUN-BOX	# OBS	SHAPIRO-WILK	LJUN-BOX	# OBS	SHAPIRO-WILK	LJUN-BOX	# OBS	SHAPIRO-WILK	LJUN-BOX	# OBS
AUSTRALIA	ARIMA(2,0,2)		44	ARIMA(3,1,2)		44	ARIMA(3,1,1)		47	ARIMA(2,2,3)		44
	0.497	0.758		0.130	0.658		0.802	0.941		0.143	0.994	
AUSTRIA	ARIMA(2,0,3)		44	ARIMA(2,1,1)		44	ARIMA(5,2,1)		33	ARIMA(3,3,5)		44
	0.914	0.985		0.627	0.922		0.664	0.993		0.067	0.806	
BELGIUM	ARIMA(3,0,4)		44	ARIMA(3,2,2)		44	ARIMA(3,1,2)		33	ARIMA(3,1,4)		44
	0.9312	0.9298		0.6561	0.9889		0.629	0.9814		0.068	0.997	
CANADA	ARIMA(2,0,3)		44	ARIMA(3,2,2)		44	ARIMA(2,1,2)		33	ARIMA(2,2,2)		44
	0.4094	0.6512		0.3094	0.8740		0.9486	0.5024		0.359	0.162	
CZECH REPUBLIC	ARIMA(4,0,2)		44	ARIMA(3,2,4)		44	ARIMA(2,1,2)		29	ARIMA(5,1,2)		38
	0.2067	0.4155		0.5861	0.2590		0.5245	0.7955		0.066	0.911	
DENMARK	ARIMA(3,0,3)		44	ARIMA(3,2,2)		44	ARIMA(0,2,3)		47	ARIMA(3,1,3)		44

⁵ As an illustrative example, consider the case of Australia, where the expected retirement duration is 19 years. Consequently, the projected values span from 2023 to 2041. The same approach is applied to the remaining countries, taking into account only the variables relevant to each pension scheme, as specified in equations (3) and (4).

	0.7099	0.9943		0.7681	0.9985		0.1026	0.8713		0.188	0.971	
FINLAND	ARIMA(3,0,3)		44	ARIMA(3,2,3)		44	ARIMA(4,1,2)		33	ARIMA(1,1,1)		44
	0.5011	0.5128		0.5811	0.9752		0.8684	0.9815		0.115	0.988	
FRANCE	ARIMA(2,0,4)		44	ARIMA(1,2,2)		44	ARIMA(2,1,1)		33	ARIMA(3,2,2)		39
	0.2908	0.9968		0.3858	0.9202		0.3206	0.7186		0.091	0.994	
HUNGARY	ARIMA(3,0,2)		44	ARIMA(1,2,1)		32	ARIMA(2,2,2)		29	ARIMA(2,1,2)		44
	0.8308	0.9071		0.2751	0.3586		0.684	0.6749		0.619	0.952	
ICELAND	ARIMA(4,0,2)		44	ARIMA(3,2,1)		44	ARIMA(5,1,2)		47	ARIMA(4,2,2)		44
	0.9494	0.9948		0.3387	0.878		0.1667	0.8787		0.233	0.836	
JAPAN	ARIMA(4,0,4)		44	ARIMA(0,2,2)		44	ARIMA(3,1,2)		33	ARIMA(3,2,2)		44
	0.1119	0.7962		0.0934	0.9964		0.7135	0.2791		0.156	0.994	
PORTUGAL	ARIMA(3,0,3)		44	ARIMA(2,1,2)		44	ARIMA(1,2,3)		29	ARIMA(3,1,3)		44
	0.5680	0.6563		0.9301	0.9015		0.1058	0.9646		0.131	0.317	
SLOVENIA	ARIMA(4,0,3)		44	ARIMA(4,2,1)		28	ARIMA(2,2,5)		29	ARIMA(3,2,2)		44
	0.9867	0.7378		0.3044	0.3953		0.2934	0.3849		0.209	0.991	
SPAIN	ARIMA(3,0,3)		44	ARIMA(2,2,1)		44	ARIMA(2,0,2)		33	ARIMA(3,2,3)		44
	0.1355	0.9933		0.5717	0.5574		0.1694	0.9248		0.381	0.934	
SWITZERLAND	ARIMA(3,0,2)		44	ARIMA(3,2,2)		44	ARIMA(2,1,2)		33	ARIMA(5,2,5)		44
	0.3432	0.9541		0.2407	0.8473		0.1703	0.9491		0.067	0.996	
TÜRKİYE	ARIMA(4,0,2)		44	ARIMA(2,1,1)		24	ARIMA(2,2,2)		29	ARIMA(2,2,2)		44
	0.2926	0.5640		0.1561	0.8600		0.1736	0.8299		0.176	0.545	
UNITED STATES	ARIMA(3,0,4)		44	ARIMA(3,1,2)		44	ARIMA(2,1,2)		33	ARIMA(4,2,3)		44
	0.3117	0.9230		0.485	0.9249		0.2888	0.9195		0.309	0.983	

Table 2: Models and p-values of Shapiro-Wilk and Ljun-Box tests for the consumer price index, the active economic population, the average salaries and the pensioned population for each country. Source: author's own work.

In general, the goodness of fit of the ARIMA models presented in Table 2 was validated using the Mean Absolute Percentage Error (MAPE) and the Mean Absolute Scaled Error (MASE). MAPE calculates the average error between actual and forecasted values. In this case, average MAPE values were approximately 27%, indicating good and acceptable fits, taking into account that the prediction periods can be up to half the length of the historical information periods used to build the models. On the other hand, MASE compares the model's mean absolute error to that of a naïve benchmark model. The ARIMA models yielded average MASE values of 26%, also suggesting a good fit. It is important to note that the number of forecasted annual observations for each model is determined by the retirement expectancy of each country, which are shown in Table 1⁶.

Although, ARIMA models could not have great predictive capacity in the long term, they only serve as a watershed to outline future behaviour of each variable. This allows us to obtain the numerator and denominator of (3), or the denominator of (4), as the case may be. Additionally, each model has different estimation settings, however, for each of them, the necessary assumptions for its application were confirmed and subsequently, the evaluation was carried out for its acceptance, to highlight the autocorrelation and normality of errors.

⁶ It is evident that the results of the proposed scoring are sensitive to the degree of error in the estimated stochastic models and to the methodology itself (ARIMA), since it is typically used to predict shorter time periods.

With all the elements described, the central axes of the model, coverage (1), adequacy (2), and sufficiency (3) or (4), are calculated for each analysed country and are showed in Table 3.

COUNTRY	COVERAGE	ADEQUACY	SUFFICIENCY
AUSTRALIA	0.6870	0.3557	0.9191
AUSTRIA	1.0000	1.0000	0.6025
BELGIUM	0.8997	0.6214	0.4531
CANADA	0.8185	0.5257	0.4003
CZECH REPUBLIC	1.0000	0.8971	0.3164
DENMARK	0.9722	1.0000	0.8484
FINLAND	1.0000	0.8343	0.4242
FRANCE	0.9698	0.8229	0.5832
HUNGARY	0.9894	0.7243	0.4042
ICELAND	1.0000	0.6157	1.0000
JAPAN	1.0000	0.4629	0.3876
PORTUGAL	0.8655	1.0000	0.3944
SLOVENIA	1.0000	0.6014	0.6368
SPAIN	0.6713	1.0000	0.7892
SWITZERLAND	0.7500	0.5700	0.6997
TÜRKİYE	0.5096	0.9850	0.8114
UNITED STATES	0.8987	0.5586	0.3937

Table 3: Coverage, adequacy and sufficiency by country, 2023. Source: author's own work.

4.2 Analysis of principal components of coverage, adequacy, and sufficiency of pension systems

Based on these data, a way to combine them is explored in such a way that they can be summarized in a single measure that describes the generic pension situation for each country. In this way, we use principal component analysis (see Annex 2) as a manner of searching for a relationship between data for each country and resume it in an only measure that allows understand and evaluate the performance of the different pensions systems around the world, which is the main objective of the paper. Although *a priori*, it can be judged, given the low-middle correlation of the variables (see Figure 1), the development of the model would be implausible, during the execution, two highly relevant premises are concluded. The first allows us to frame that, by having the same number of original components and variables, there are essential elements with a different approach for the explanation of the phenomenon under study. The second argument focuses on taking advantage of the intermediate results obtained in the principal component analysis.

The Pearson correlation between the variables, coverage, adequacy and sufficiency, of Table 3 is situated at levels very close to zero, except for the coverage and sufficiency, where the heatmap highlights a middle inverse correspondence between them (-0.4) (see Figure 1). It is important to note that we also compute the Spearman correlation and the differences between both methods are not significative.

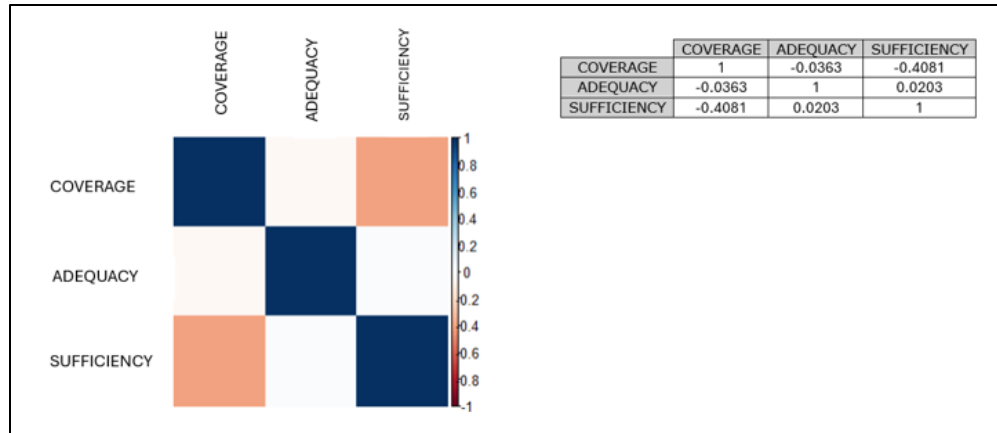


Figure 1: Correlation between the coverage, adequacy and sufficiency variables. Source: author's own work.

With the principal component analysis executed in the R software, specifically with the FactoMineR package (Lê *et al*, 2008) and the PCA function, three principal components are obtained from the coverage, adequacy and sufficiency variables of Table 3. The percentages of explained variance for components one, two, and three are 45.70%, 40.15%, and 14.15%, respectively. Visually, Figure 2 concentrates the results. The variance explained by the first two components accumulates 85.9%, although, as it will be shown later, it incorporates to a greater extent only two original variables (adequacy and sufficiency) and the third component accounts for nearly 85% of the remaining variance, primarily associated with coverage).

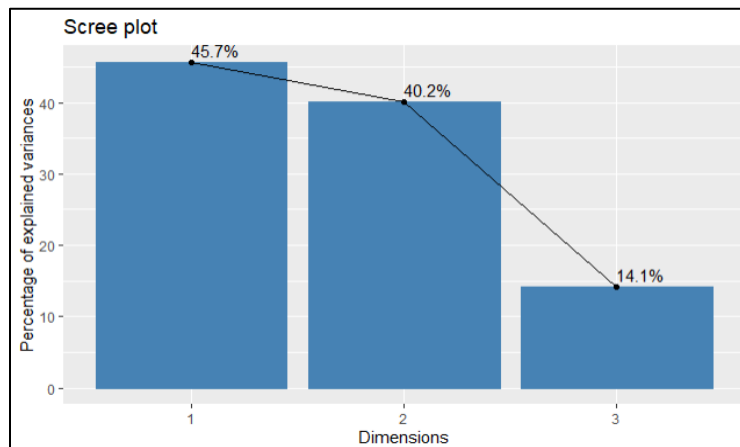


Figure 2: Explained variance by the three principal components of coverage, adequacy and sufficiency variables. Source: author's own work.

The next part corresponds to the relationship of the variables and the components; Figure 3 shows the behaviour of the two components with the greatest explained variance. The components with the greatest explained variance show some prominent patterns, such as the location of the most countries with a defined benefit pension system situated in the left.

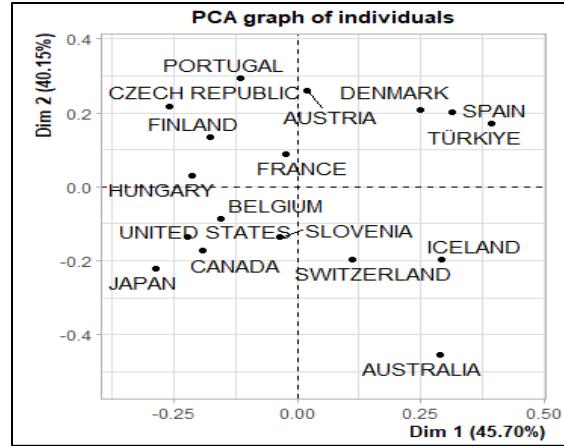


Figure 3: Graph of the countries for the two first components. Source: author's own work.

Consecutively, Figure 4 summarizes the contribution of the variables to each of the principal components based on the linear combinations, according to the importance given to each original variable. These results are essential for the construction of the evaluation tool, in conjunction with the explained variance of the components, which will be the weight that will be given to each of these new variables.

	X1	X2	X3
COVERAGE	15.1430%	0.4015%	84.4554
ADEQUACY	4.5398%	95.4079%	0.0522%
SUFFICIENCY	80.3171%	4.1905%	15.4923%

Figure 4: Contribution of the coverage, adequacy and sufficiency variables to the components⁷. Source: author's own work.

4.3 Proposal for a pension system evaluation technique

The reason for executing this analysis was not so focused on dimensionality reduction, but rather, in a way to weight the variables in a non-uniform manner, that is, not to assign the same weight equally to the variables. In Chao and Wu (2017) and Broby and Smyth (2025), are presented some ideas for constructing an index follow reasoning similar to the proposed in this paper but applied to different professional fields. Likewise, the proposed methodology takes advantage of the benefits inherent in principal component analysis, resulting in an aggregate score that retains the information from all the variables. Therefore, the pension classification score is defined as follows:

$$\text{Score} = 0.4570 \cdot X1 + 0.4015 \cdot X2 + 0.1415 \cdot X3. \quad (5)$$

Where the multiplication is weighted by the contribution made by each variable (see Figure 4), such that:

$$X1 = 0.15143 \cdot \text{Coverage} + 0.04539 \cdot \text{Adequacy} + 0.80317 \cdot \text{Sufficiency}. \quad (6)$$

$$X2 = 0.00401 \cdot \text{Coverage} + 0.95407 \cdot \text{Adequacy} + 0.04190 \cdot \text{Sufficiency}. \quad (7)$$

$$X3 = 0.84455 \cdot \text{Coverage} + 0.00052 \cdot \text{Adequacy} + 0.15492 \cdot \text{Sufficiency}. \quad (8)$$

⁷ The contribution of each variable to the component is defined by the equation $\frac{\cos^2(\theta_{ij})}{\sum_j \cos^2(\theta_{ij})}$, where θ_{ij} is the angle between the variable and the component.

By substituting (6), (7), and (8) into (5), we obtain the final weights based on the original variables:

$$Score = 0.19032 \cdot Coverage + 0.40388 \cdot Adequacy + 0.40579 \cdot Sufficiency. \quad (9)$$

With these weights, the following scores are obtained for each country (see Table 4), with a mean of 0.70, and the minimum and maximum scores are 0.53 and 0.93 respectively. Moreover, 59% of countries are ranked below average, and none reaches the maximum value of 1.

COUNTRY	SCORE
AUSTRALIA	0.6474
AUSTRIA	0.8387
BELGIUM	0.6061
CANADA	0.5306
CZECH REPUBLIC	0.6810
DENMARK	0.9332
FINLAND	0.6994
FRANCE	0.7536
HUNGARY	0.6448
ICELAND	0.8448
JAPAN	0.5345
PORTUGAL	0.7287
SLOVENIA	0.6916
SPAIN	0.8519
SWITZERLAND	0.6569
TÜRKİYE	0.8241
UNITED STATES	0.5564

Table 4: Final scores according to the proposal technique for each country. Source: author's own work.

4.4 Complementary support to the classification of the pension systems with conglomerates

To obtain a general view of the situation of the countries, a clustering analysis (see Annex 2) is carried out with the NbClust package (Charrad *et al*, 2014 and 2022) considering the coverage, adequacy, and sufficiency of the pension systems shown in Table 3, as a complementary way to support the principal component analysis.

Specifically, as mentioned in the previous section, Ward's method is used, since this methodology aims to maximize homogeneity within the clusters (Aldás & Uriel, 2017). The efficient number of clusters resulted in five, shown in the dendrogram in Figure 5, where the red line indicates the separation of the different groups obtained:

Group 1: Belgium, Canada, Japan, Slovenia, United States.

Group 2: Australia, Iceland, Switzerland.

Group 3: Spain, Türkiye.

Group 4: Austria, Denmark, France.

Group 5: Czech Republic, Finland, Hungary, Portugal.

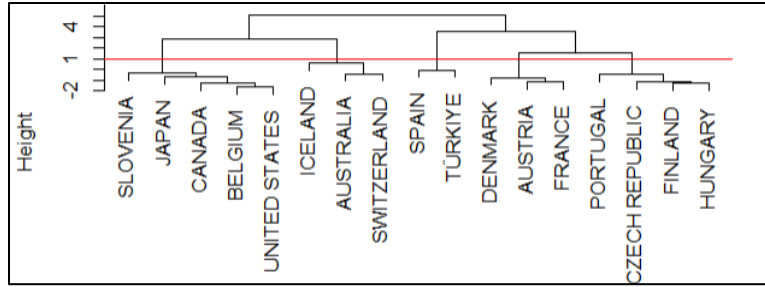


Figure 5: Clusters of countries according to the coverage, adequacy and sufficiency variables. Source: author's own work.

Additionally, the following table summarizes the average, minimum, and maximum values of the original variables and the scores of the previously defined groups. Within each group, it is possible to identify certain patterns that explain the resulting clusters.

GROUP	STATISTICS	COVERAGE	ADEQUACY	SUFFICIENCY	SCORE
1	AVERAGE	0.9234	0.5540	0.4543	0.5838
	MIN	0.8185	0.4629	0.3876	0.5306
	MAX	1.0000	0.6214	0.6368	0.6916
2	AVERAGE	0.8123	0.5138	0.8729	0.7164
	MIN	0.6870	0.3557	0.6997	0.6474
	MAX	1.0000	0.6157	1.0000	0.8448
3	AVERAGE	0.5905	0.9925	0.8003	0.8380
	MIN	0.5096	0.9850	0.7892	0.8241
	MAX	0.6713	1.0000	0.8114	0.8519
4	AVERAGE	0.9807	0.9410	0.6780	0.8418
	MIN	0.9698	0.8229	0.5832	0.7536
	MAX	1.0000	1.0000	0.8484	0.9332
5	AVERAGE	0.9637	0.8639	0.3848	0.6885
	MIN	0.8655	0.7243	0.3164	0.6448
	MAX	1.0000	1.0000	0.4242	0.7287

Table 5: Comparison of statistics by group. Source: author's own work.

5. DISCUSSION

Figure 6 concentrates the scores obtained from the proposed evaluation technique (9) and are ordered according to the punctuation such that the results can be understood easier. However, some generic aspects of the methodology will be discussed additionally to explain the scores obtained by each country.

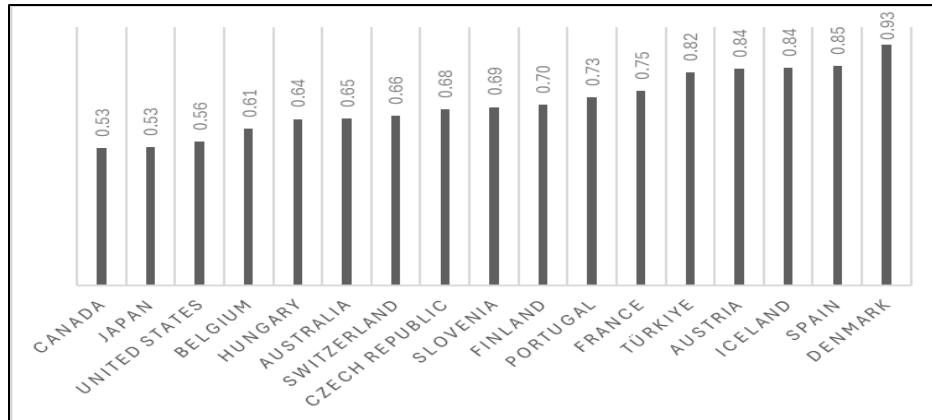


Figure 6: Ordered scores for the countries. Source: author's own work.

Figures 7, 8, and 9 contain the values of each axe of the classification technique (24): coverage (1), adequacy (2), and sufficiency (3) or (4), respectively. The average coverage is 0.88, with observations located between 0.50 and 1.00, besides, 50% of the countries have a score above 0.96. For the adequacy the average is 0.74, with observations located between 0.35 and 1.00. Finally, the average sufficiency is 0.59, with observations located between 0.31 and 1.00 and eight countries have a score below 0.5.

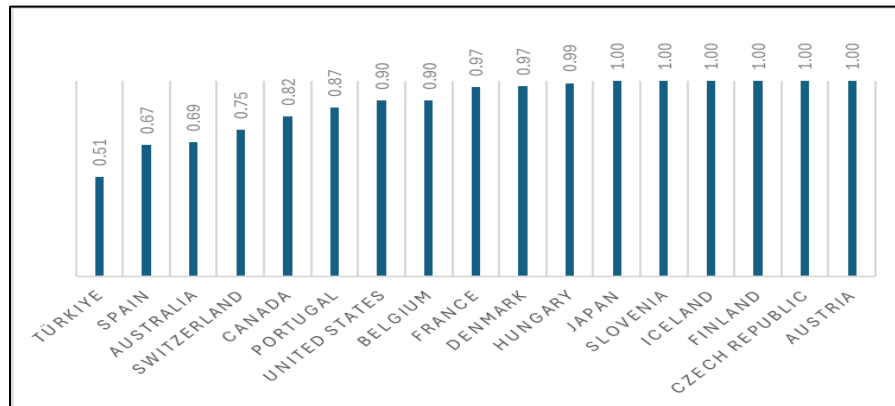


Figure 7: Coverage for the countries. Source: author's own work.

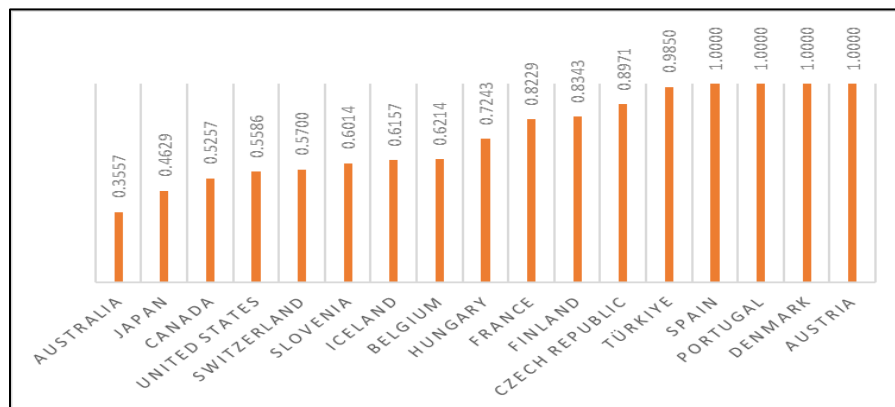


Figure 8: Adequacy for the countries. Source: author's own work.

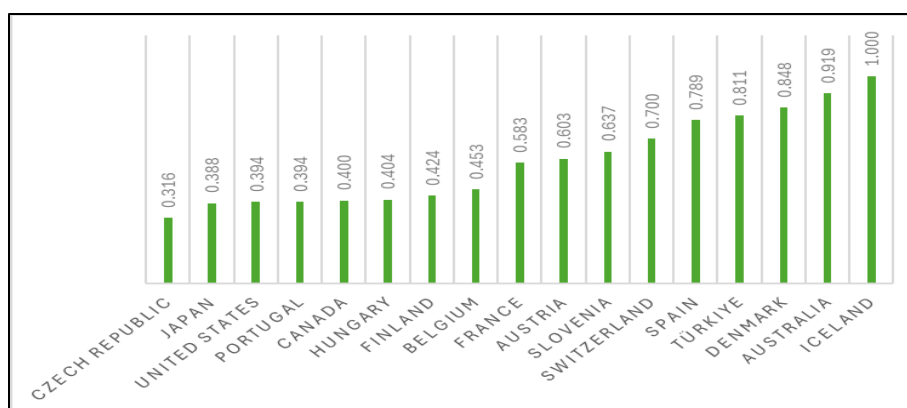


Figure 9: Sufficiency for the countries. Source: author's own work.

In this sense, the country with the highest score obtained is Denmark with 0.93, followed by Spain and Iceland with an approximately score of 0.85. With this, it can be expressed that countries with a defined contribution system within the study, achieved the best scores.

In fact, Australia, the other country with a defined contribution system, has very good sufficiency conditions, however, its score was affected due to it having the lowest replacement rate of all countries. Likewise, in the dendrogram, Denmark and Austria are part of the same group along with France, as a result of having scores above average in the three axes.

Furthermore, Spain takes the second place in the ranking. The level of adequacy and sufficiency it has is great, and, although the level of coverage is the second lowest, there is a large change of 16 points compared to Türkiye which ranked last place in this same axe. For this last reason, Spain and Türkiye are grouped in the same conglomerate.

The next grouping is made up of the Czech Republic, Finland, Hungary, and Portugal. In general, these countries occupy good positions in one aspect, as well as not-so-good positions in complementary elements, which is why they are located around the average value of the classification scores.

Finally, the last group is made up of Slovenia, Japan, Canada, Belgium, and the United States, and it is characterized for having obtained the lowest scores, which is not surprising given the scores that these countries attained at the individual level in coverage, adequacy, and sufficiency. Canada and Japan are the countries with the lowest score, and, although they did not obtain the lowest level in any of the elements, they were surpassed by the rest of the elements of those countries that were at low levels of coverage, adequacy or sufficiency.

Although, given the coverage, adequacy, and sufficiency scores, it can be understood why each of the countries obtained their respective score in the ranking; the support of the Mercer index scores is added in Table 6. This is an index composed of three sub-indexes: adequacy, sustainability, and integrity (Mercer, 2023). Consistent with the obtained results, the Mercer index ranks Iceland and Denmark in the top three. On the contrary, Japan has one of the lowest scores according to the Mercer index, as in the model of this analysis. The case of Türkiye stands out, which in the Mercer index has a lower level than Japan. One of the causes of this difference is the greater incorporation of variables in Mercer index, however, since the projections made in the analysis of ARIMA models, it was observed that Türkiye will present problems in the long term due to the growth of its population, it is an element that has been taking into account in a more extended way in the Mercer index. For the rest of the countries, the behaviour seems quite similar, although with some variations in the positions, which is explained by the approach applied in each analysis. The following table presents the Mercer Index scores alongside the proposed scores from this paper. It is important to note that the Pearson and Spearman correlation coefficients were computed, and in both cases, the

values were close to 0.05. This result is largely due to significant variations in countries such as Austria, Canada, and Türkiye.

9 á ∂ Ä ± • ”	~ E • 9 E • € Ä ? E “	ç • á ç á € ? € 9 á • E Ä □
∂ € • xf	77	65
∂ € • f	53	84
7 Ex] f ∂ ~	69	61
9 Ä ?	70	53
9 › E 9 c ç E ç ∂ 7 x f 9	NA	68
? E Ä ~ • u	81	93
[f Ä x Ä ?	77	70
[• Ä 9 E	62	75
c ∂ Ä] • ”	NA	64
∂ Ex Ä ?	84	84
s ç Ä	56	53
ç á • ± ∂] x	67	73
€ á . E Ä f	NA	69
€ f Ä	62	85
€ ∂ E • x Ä ?	72	66
± • u f “ E	46	82
∂ Ä f E ? € ± E €	63	56

Table 6: Comparative analysis of the Mercer Index and the proposed score, 2023. Source: author’s own work.

Figure 10 displays, in a two-dimensional space, the values of the countries in the Mercer Index and in the proposed score, as a complement to Table 6. If we leave out Iceland and Denmark, a certain degree of non-linear correlation can be identified between the Mercer Index and the proposed score (under this condition we have a Spearman correlation coefficient of -0.41).

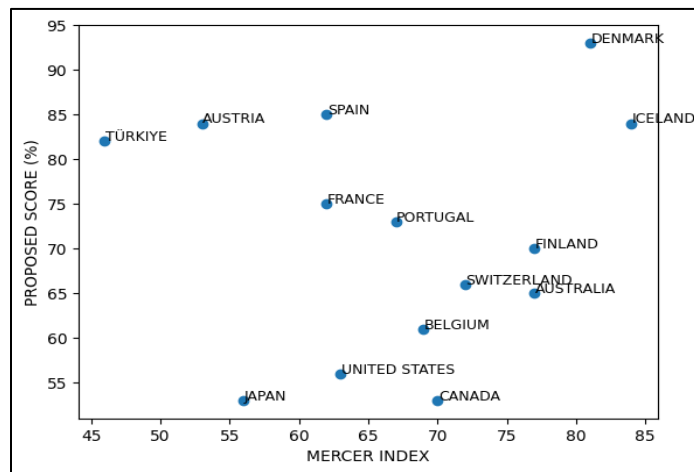


Figure 10: Mercer Index and the proposed score, 2023. Source: author’s own work.

In addition, the results of this study are comparable to the synthetic indicator proposed by Alonso-Fernández *et al* (2018) for the countries included in both studies, despite the fact that Alonso-Fernández *et al* indicator was defined for the year 2018 (see Table 7). Although they do not address the same elements completely, since in this study we define adequacy from a simpler point of view and combine it with coverage and financial sufficiency. As for the comparative results, Spain and Denmark are in the highest positions in both cases, France, Portugal, Finland and Slovenia are also located in the intermediate positions and Belgium occupies the lowest positions.

9 á ∂Ä±”	xá ÄÇá ſE• Ä Ä? E• E± x fÄ? 9 ±á• Ç□□□	ç• á çá ÇE? Ç9 á • EÇ□□Ç □
∂Ç• xf	NA	65
∂Ç• f	55.5	84
7Ex] f0~	NA	61
9 Ä ?	NA	53
9>E9 c ç Eç∂7x9	59.6	68
?EÄ~ •u	67.3	93
[fÄx Ä?	54.6	70
[• Ä9E	59.9	75
c ∂Ä] •”	66	64
9Ex Ä?	NA	84
s ç Ä	NA	53
çá•±∂] x	54.4	73
Çxá .EÄf	53.3	69
Çç fÄ	69	85
ÇÖ f±E• x Ä?	NA	66
±ſ •uf•E	NA	82
∂Äf±E? ÇÇ ±EÇ	NA	56

Table 7: Comparative analysis of the Alonso-Fernández *et al* indicator and the proposed score. Source: author’s own work.

Similarly to the previous comparison, Figure 11 shows in a two-dimensional space the values of the countries in the Alonso-Fernández *et al* indicator and in the proposed score. In this case, the Spearman correlation coefficient is approximately 0.42. However, when Denmark and Spain are excluded from the analysis, the coefficient drops to -0.21, which may be attributed to the temporal differences between the two analyses.

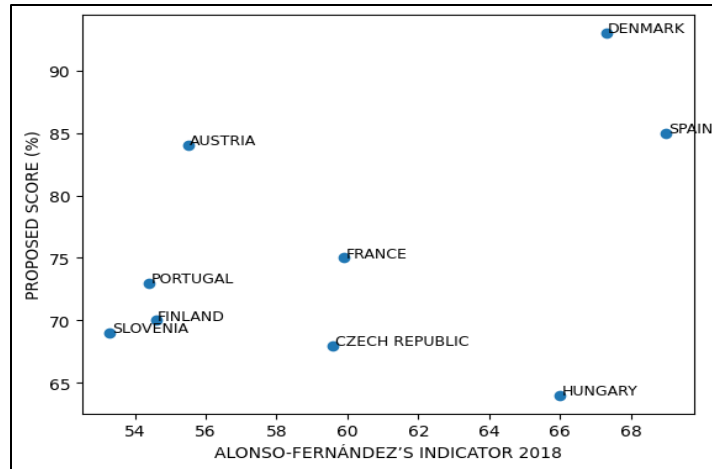


Figure 11: Alonso-Fernández *et al* indicator and the proposed score. Source: Author's own work.

6. CONCLUSIONS

The results of the intermediate processes in the new proposed classification technique, allow us to extract valuable arguments for the identification of improvement opportunities that countries present in terms of their pension system. As it was shown in this study and in the majority of pension case studies, the financial aspect turns out to be the fundamental piece to take into account; although defined contribution systems seem to have an advantage in this study in terms of sufficiency, this could be modified if, for defined benefit systems, government contributions were included, which can also appear in some defined contribution systems when the minimum funds are not reached.

The previous argument leads to considering the other two elements, adequacy and coverage. The original measures, by themselves, are very useful for governments, since they can identify those aspects in which there are improvement plans. This would not be limited only to identifying actions to improve, given that evaluations can be carried out yearly and allow analysing the effectiveness of the changes made to the pension systems. In this context, the proposed methodology is intended as a complement rather than a replacement for existing indexes such as the Mercer Index or the Alonso-Fernández *et al* indicator. As shown in Figures 10 and 11, there is a certain degree of correlation between the proposed score and each of the other two indicators, respectively, which highlights the distinct approach of the proposed score

Additionally, it highlights the expansion that this analysis could have, in terms of the conceptual and structural standings already mentioned of the coverage, adequacy or financial sufficiency, but also, in the number of countries that are part of the study. Although the data used is not difficult to determine, nor does it contain sensitive information, the repositories found to this point are few. In the same sense, once the data is available, it would be of great value to be able to make a clear distinction between them for those countries that carried out structural reforms in their pension systems and that currently have more than one pension scheme, all this with the purpose of analysing the very particular behaviour of these regions.

Finally, once the results have been analysed, the governments of each country will find the necessary actions to improve those points that can be overcome and ensure the improvement of social well-being. However, it is essential to continue encouraging on a personal level a culture of awareness and promotion of savings in old age, particularly in developing countries which are, in general terms, the ones that find the most areas of opportunity in this regard as a consequence of an unsubstantial

financial education, while countries should establish target scores to guide the improvement of their pension systems.

7. ACKNOWLEDGEMENTS

We would like to express our sincere gratitude to Dr. Eva Boj del Val for her valuable support.

8. REFERENCES

- Ablanedo, E. & Barón, A. (2020). Análisis de los sistemas de pensiones europeos e internacionales. Iniciativa de la Unión Europea para la protección social, el trabajo y el empleo. https://socieux.eu/wp-content/uploads/2020/06/An%C3%A1lisis_de_los_Sistemas_de_Pensiones.pdf
- Aldás, J. & Uriel, E. (2017). Análisis multivariante aplicado con R. Ediciones Paraninfo. Madrid, España.
- Alonso-Fernández, J.-J., Meneu-Gaya, R., Devesa-Carpio, E., Devesa-Carpio, M., Domínguez-Fabian, I. & Encinas-Goenechea, B. (2018). From the Replacement Rate to the Synthetic Indicator: A Global and Gender Measure of Pension Adequacy in the European Union. *Social Indicators Research*, 138(1), 165–186. <https://www.jstor.org/stable/48715115>
- Barr, N. & Diamond, P. (2006). The economics of pensions. *Oxford Review of Economics Policy*, 22.
- Biggs, A. & Springstead, G. (2008). Alternate measures of replacement rates for social security benefits and retirement income. *Social Security Bulletin*, 68(2), 1–19. PMID: 19102135.
- Bodie, Z., Shoven, J. B., & Wise, D. A. (1988). Pensions in the U.S. Economy. University of Chicago Press.
- Box, G. & Cox, D. (1964). An analysis of transformations (with discussion). *Journal of the Royal Statistical Society, Series B*, 26, 211–252.
- Box, G. Jenkins, G., Reinsel, G. & Ljung, G. (2016). Time series Analysis Forecasting and Control (5). Wiley.
- Broby, D., & Smyth, W. (2025). On the use of principal components analysis in index construction. <https://systems.enpress-publisher.com/index.php/FSJ/article/view/10858/5404>
- Brockwell, P. & Davis, R. (2002). Introduction to Time Series and Forecasting (2). Springer.
- Chao, Y. & Wu, C. (2017). Principal component-based weighted indices and a framework to evaluate indices: Results from the Medical Expenditure Panel Survey 1996 to 2011. *PLoS ONE*, 12(9), e0183997. <https://doi.org/10.1371/journal.pone.0183997>
- Charrad, M., Ghazzali, N., Boiteau, V. & Niknafs, A. (2014). NbClust: An R Package for Determining the Relevant Number of Clusters in a Data Set. *Journal of Statistical Software*, 61(6), 1–36. <https://www.jstatsoft.org/v61/i06/>
- Charrad, M., Ghazzali, N., Boiteau, V. & Niknafs, A. (2022). NbClust: An R Package for Determining the Relevant Number of Clusters in a Data Set. R package version 3.0.1. <https://CRAN.R-project.org/package=NbClust>

- Chybalski, F. (2015). The Multidimensional Efficiency of Pension System: Definition and Measurement in Cross-Country Studies. *Social Indicators Research*, 128, 15–34. <https://doi.org/10.1007/s11205-015-1017-3>
- Chybalski, F. & Marcinkiewicz, E. (2016). The Replacement Rate: An Imperfect Indicator of Pension Adequacy in Cross-Country Analyses. *Social Indicators Research*, 126, 99–117. <https://doi.org/10.1007/s11205-015-0892-y>
- Clark, R., Craig, L. & Wilson, J. (2003). A History of Public Sector Pensions in the United States. Pension Research Council, The Wharton School of the University of Pennsylvania.
- Cole, C. & Liebenberg, A. (2008). An examination of retirement income adequacy measures and factors affecting retirement preparedness. Available at SSRN <http://ssrn.com/abstract=1272067> or <http://dx.doi.org/10.2139/ssrn.1272067>
- Conde-Ruíz, J. & González, C. (2018). European pension system: Bismarck or Beveridge?. Fundación de Estudios de Economía Aplicada. <https://documentos.fedea.net/pubs/dt/2018/dt2018-01.pdf>
- Contreras, C. (2020). Modelo deseable para un sistema de pensiones igualitario, justo y sostenible. Conferencia Interamericana de Seguridad Social. <https://ciss-bienestar.org/cuadernos/pdf/modelo-deseable-para-un-sistema-de-pensiones-igualitario-justo-y-sostenible.pdf>
- Cowpertwait, P. & Metcalfe, A. (2009). Introductory Time Series with R. Springer.
- Das, J. (2015). Business Forecasting: Box-Jenkins Methodology. University of Calcutta. https://www.researchgate.net/publication/342551929_Business_Forecasting_Box-Jenkins_Methodology
- Devesa-Carpio, E., Rosado-Cebrian, B. & Álvarez-García, J. (2020). Sustainability of Public Pension Systems, Springer Books, in: Peris-Ortiz, M., Álvarez-García, J., Domínguez-Fabián, I., Devolder, P. (ed.), Economic Challenges of Pension Systems, chapter 8, pages 125-154, Springer.
- Fernández, P. (2003). El Aerarium Militare. Espacio, Tiempo y Forma, Serie II, Historia Antigua, 16, 197-214.
- Gallagher, N., Sullivan, D. & Palacios, M. (2020). The Effect of Data Centering on PCA Models. Eigenvector Research Incorporated. <https://eigenvector.com/wp-content/uploads/2020/06/EffectofCenteringonPCA.pdf>
- Güven, U. & Holzmann, R. (2009). Adequacy of Retirement Income after Pension Reforms in Central, Eastern, and Southern Europe: Eight Country Studies. Directions in Development; finance. World Bank. <http://hdl.handle.net/10986/2610>
- Husson, F., Lê, S. & Pagès, J. (2011). Exploratory Multivariate Analysis by Example Using R. CRC press.
- Hyndman, R.J. & Khandakar, Y. (2008). Automatic time series forecasting: the forecast package for R. *Journal of Statistical Software*, 27(3), 1–22. doi:10.18637/jss.v027.i03

- Hyndman, R., Athanasopoulos, G., Bergmeir, C., Caceres, G., Chhay, L., O'Hara-Wild, M., Petropoulos, F., Razbash, S., Wang, E. & Ysmeen, F. (2024). forecast: Forecasting functions for time series and linear models. R package version 8.23.0. <https://pkg.robjhyndman.com/forecast/>.
- Jiménez, L. & Cuadros J. (2003). Evaluación de las reformas a los sistemas de pensiones: cuatro aspectos críticos y sugerencias de políticas. Unidad de Estudios Especiales, Secretaría Ejecutiva: Comisión Económica para América Latina y el Caribe. <https://repositorio.cepal.org/server/api/core/bitstreams/5fc288c1-eff5-41fb-b368-ef1ca178b142/content>
- Knoef, M., Been, J., Alessie, R., Caminada, K., Goudswaard, K. & Kalwij, A. (2016). Measuring retirement savings adequacy: developing a multi-pillar approach in the Netherlands. *Journal of Pension Economics and Finance*, 15(1), 55–89. <https://doi.org/10.1017/S1474747214000341>
- Lê S., Josse J. & Husson F (2008). FactoMineR: A Package for Multivariate Analysis. *Journal of Statistical Software*, 25(1), 1–18.
- Meneu-Gaya, R., Encinas-Goenechea, B. & Domínguez-Fabián, I. (2020). Equity of Public Pension System, Springer Books, in: Peris-Ortiz, M., Álvarez-García, J., Domínguez-Fabián, I., Devolder, P. (ed.), *Economic Challenges of Pension Systems*, chapter 8, pages 155-172, Springer.
- Mercer. (2023). Índice Global de Pensiones de Mercer CFA Institute 2023. <https://www.mercer.com/es-ar/insights/investments/market-outlook-and-trends/mercercfa-global-pension-index/>
- Mesa-Lago, C. (2020). Evaluation of Four Decades of Pension Privatization in Latin America, 1980-2020: Promises and Reality. SSRN Electronic Journal. https://papers.ssrn.com/sol3/Delivery.cfm/SSRN_ID3812430_code2517124.pdf?abstractid=3812430&mirid=1
- Mesa-Lago, C. (2022). Pensiones de capitalización individual en América Latina: efectos, reformas, impacto del COVID-19 y propuestas de política. *Políticas Sociales. Comisión Económica para América Latina y el Caribe*. <https://repositorio.cepal.org/server/api/core/bitstreams/c973ffd4-2bf6-4c4c-912b-49c3c8d5cedd/content#:~:text=Los%20pa%C3%ADses%20tratados%20son%20Bolivia,Rep%C3%ABlica%20Dominicana%20y%20el%20Uruguay.>
- Palmer, B. (1989). Tax Reform and Retirement Income Replacement Ratios. *The Journal of Risk and Insurance*, 56(4), 702-725.
- Pokorný, J. (2020). Evaluation of pension sustainability and its application. *Economic Spectrum*, 15.
- R Development Core Team (2024). R: The R project for statistical computing. Vienna (Austria). <http://www.R-project.org/>
- Rosado-Cebrian, B., Peris-Ortiz, M. & Rueda-Armengot, C. (2020). Adequacy of Public Pension Systems, Springer Books, in: Peris-Ortiz, M., Álvarez-García, J., Domínguez-Fabián, I., Devolder, P. (ed.), *Economic Challenges of Pension Systems*, chapter 8, pages 173-191, Springer.

- Scheubel, B. (2013). Bismarck's Pension System. Bismarck's Institutions: A Historical Perspective on the Social Security Hypothesis (pp.77-104). Mohr Siebeck GmbH and Co. KG
- Shumway, R. & Stoffer, D. (2006). Time Series analysis and Its applications With R Examples (2). Springer.
- Social Security Administration. (2023). International Update: Recent Developments in Foreign Public and Private Pensions (International Update). Office of Policy, Office of Research, Evaluation, and Statistics, U.S. Social Security Administration. https://www.ssa.gov/policy/docs/progdesc/intl_update/2023-03/index.html
- Subsecretaría de Previsión Social. (2024). Sistema de Pensiones, Pilares del sistema de pensiones. Subsecretaría de Previsión Social: Gobierno de Chile. <https://previsionsocial.gob.cl/seguridad-social/sistema-de-pensiones/#:~:text=El%20sistema%20de%20pensiones%20chileno,de%20propiedad%20de%20cada%20trabajador.>
- United Nations. (2017). World Population Prospects The 2017 Revision. Department of Economic and Social Affairs, Population Division. https://population.un.org/wpp/publications/files/wpp2017_keyfindings.pdf
- World Bank Group (1994). Averting the old age crisis: policies to protect the old and promote growth. World Bank Group: Washington, D.C. <http://documents.worldbank.org/curated/en/973571468174557899/Averting-the-old-age-crisis-policies-to-protect-the-old-and-promote-growth>
- World Bank Group (2008). The World Bank pension conceptual framework. World Bank pension reform primer series. World Bank Group: Washington, D.C. <http://documents.worldbank.org/curated/en/389011468314712045/The-World-Bank-pension-conceptual-framework>
- World Health Organization. (2022). Ageing and health. World Health Organization: Newsroom. <https://www.who.int/news-room/fact-sheets/detail/ageing-and-health>
- Yuh, Y. (2011). Assessing Adequacy of Retirement Income for U.S. Households: A Replacement Ratio Approach. The Geneva Papers on Risk And Insurance - Issues And Practice, 36(2), 304-323. <https://doi.org/10.1057/gpp.2011.7>

Annex 1. Pensioned population sources (searched in April 2024).

Country	Source
Australia	https://www.aihw.gov.au/reports/australias-welfare/income-support-older-australians#How_many_older
Austria	https://www.statistik.at/en/statistics/population-and-society/social-benefits/pensions/pensions
Belgium	https://www.statista.com/statistics/542467/number-of-pensioners-in-belgium-by-age/
Canada	https://www.osfi-bsif.gc.ca/en/oca/actuarial-reports/actuarial-report-31st-canada-pension-plan
Czech Republic	https://www.cssz.cz/documents/20143/99587/2022_Statistick%C3%A1%20ro%C4%8Denka%20z%20oblasti%20d%C5%AFchodov%C3%A9ho%20poji%C5%A1t%C4%9Bn%C3%AD%20za%20rok%202022.pdf/c4b3c136-17c2-8913-8a24-dafda6bc60bf
Denmark	https://www.dst.dk/en/Statistik/emner/sociale-forhold/offentligt-forsoergede/folke-ogfoertidspensionister#:~:text=Number%20of%20recipients%20of%20national%20old%2Dage%20pension%20in%20Denmark&text=Range%3A%201040000%20to%201080000.
Finland	https://www.julkari.fi/bitstream/handle/10024/145447/statistical-yearbook-of-pensioners-in-finland-2021.pdf?sequence=1&isAllowed=y
France	https://www.statista.com/statistics/949679/general-plan-pensioners-by-gender-france/
Hungary	https://www.ksh.hu/stadat_files/szo/en/szo0054.html
Iceland	https://www.statice.is/publications/news-archive/social-affairs/pension-beneficiaries-2021/#:~:text=In%20December%202021%2C%20about%2065,group%2C%2065%20to%2066%20years.
Japan	https://www.ipss.go.jp/ssj-db/e/ssj-db-top-e.asp
Portugal	https://www.pordata.pt/en/db/portugal/search+environment/table
Slovenia	https://www.stat.si/StatWeb/en/News/Index/11251
Spain	https://www.seg-social.es/wps/wcm/connect/wss/e472fa03-a82b-4ac3-93d7-d01559e257ba/202203_Avance+provincial.pdf?MOD=AJPERES&CONVERT_TO=linktext&CACHEID=ROOTWOKSPACE.Z18_2G50H38209D640QTQ570VB2000-e472fa03-a82b-4ac3-93d7-d01559e257ba-n.0jun
Switzerland	https://www.bfs.admin.ch/bfs/en/home/statistics/social-security/occupational-pension-plans/beneficiaires-benefits.html
Türkiye	https://data.tuik.gov.tr/Bulten/Index?p=Social-Protection-Statistics-2020-37193&dil=2#:~:text=While%20the%20number%20of%20old,million%20747%20thousand%20in%202019.
United States	https://www.statista.com/statistics/194295/number-of-us-retired-workers-who-receive-social-security/#:~:text=The%20number%20of%20retired%20workers,to%20continue%20into%20the%20future.

Annex 2. Mathematical concepts.

Time series

Box *et al* (2016), Cowpertwait and Metcalfe (2009), Shumway and Stoffer (2006), along with Brockwell and Davis (2002), define a time series as a sequence of observations ordered in time and with the same periodicity, thus, analyses of these data focus on techniques that look for dependencies between observations to forecast the future value of data series using current and past values. The basic idea of the stochastic model of a time series z_t is based on successive values that assume high dependence, which is generated from independent random shocks a_t with fixed distribution, usually assumed normal with zero mean and variance σ_a^2 .

A sequence of independent random variables $a_t, a_{t-1}, a_{t-2}, \dots$ is known as a white noise process, there by, if a linear transformation is applied, then:

$$z_t = \mu + a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \dots = \mu + \psi(B)a_t, \quad (10)$$

where B is known as the delay operator ($Bz_t = z_{t-1}$) and μ is a parameter that determines the level of the process. Additionally, the sequence of weights $\psi_1, \psi_2, \dots$ can be finite or infinite and can be absolutely summable, such that $\sum_{j=0}^{\infty} |\psi_j| < \infty$. If the process meets this property, it is said to be a stationary stable process and the parameter μ is the average of the process; otherwise, the process is non-stationary and μ has no concrete meaning.

Particularly, an autoregressive model has the characteristic that the current value of the process is determined as a finite linear aggregate of previous values of the process and a random shock a_t . In this way, the values of the process at time $t, t-1, t-2, \dots$ are denoted as $z_t, z_{t-1}, z_{t-2}, \dots$ and the deviations of the process are $\bar{z}_t = z_t - \mu$, such that:

$$\bar{z}_t = \phi_1 \bar{z}_{t-1} + \phi_2 \bar{z}_{t-2} + \dots + \phi_p \bar{z}_{t-p} + a_t, \quad (11)$$

is a p order autoregressive process (AR). It also can be written as:

$$(1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p) \bar{z}_t = \phi_p(B) \bar{z}_t = a_t. \quad (12)$$

Another important model in time series is the process known as moving averages, where $\bar{z}_t$ is taken as an independent value of a finite series of q previous random shocks a . So:

$$\bar{z}_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q}, \quad (13)$$

or:

$$\bar{z}_t = (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q) a_t = \theta_q(B) a_t. \quad (14)$$

With the combination of the two previous models, the autoregressive moving average models (ARMA) are developed and are expressed mathematically as:

$$\phi_p(B) \bar{z}_t = \theta_q(B) a_t. \quad (15)$$

However, Box *et al* (2016) mention that most phenomena, both in the industrial sector and in the business field, do not present stationarity, so it is necessary to introduce the difference operator $\nabla^d = (1 - B)^d$, which satisfies $\nabla^d \bar{z}_t = \nabla^d z_t$. In this way, a model that is not stationary can be transformed as follows to achieve stationarity:

$$\phi_p(B) \nabla^d z_t = \phi_p(B) w_t = \theta_q(B) a_t \quad (16)$$

where $w_t = (1 - B)^d z_t$.

With this adhesion, the integrated autoregressive models of moving averages, ARIMA, of order (p, d, q) are defined. Likewise, for seasonal time series, with period s , ARIMA models can be extended with the operator $\nabla_s = 1 - B^s$, such that:

$$\phi_p(B)\Phi_p(B^s)\nabla^d\nabla_s^D z_t = \theta_q(B)\Theta_q(B^s)a_t. \quad (17)$$

Note that $\Phi_p(B^s)$, ∇_s^D and $\Theta_q(B^s)$ are seasonal operators and the general models are expressed as $ARIMA(p, d, q) \times (P, D, Q)_s$.

Additionally, in the previous development, it is considered that the time series data z_t has a regular behavior in which the dispersion concerning the mean is constant, that is, the series is stationary in variance. This condition can be achieved by the Box-Cox transformation, which is defined by Box and Cox (1964) as follows:

$$y^{(\lambda)} = \begin{cases} y^\lambda & \text{si } \lambda \neq 0 \\ \log y & \text{si } \lambda = 0. \end{cases} \quad (18)$$

For the time series model, consider an original data series x_t , such that:

$$z_t = \begin{cases} x_t^\lambda & \text{si } \lambda \neq 0 \\ \log x_t & \text{si } \lambda = 0. \end{cases} \quad (19)$$

For a detailed analysis of the estimation process, it is recommended to see Box and Cox (1964). Subsequently, the Box-Jenkins methodology is applied, which consists of 4 steps: identification of the process, estimation of parameters, verification of results, and forecast (Das, 2015). The validation for the ARIMA models in this paper is based on the normality (Shapiro-Wilk) and non-correlation (Ljun-Box) tests applied to residuals.

Principal Component Analysis

The next topic reviewed is principal component analysis. Aldás and Uriel (2017), as well as Husson *et al* (2011), agree in defining this multivariate technique as a process through which a set of uncorrelated and orderable variables is obtained according to the information they contain. It is important to emphasize that the principal components obtained with this analysis are the result of linear combinations of the original variables.

Mathematically, consider a sample of size n of p variables, $X_1, X_2, \dots, X_p$, the components, Z_{1i} , are expressed as a line combination, such that:

$$\begin{bmatrix} Z_{11} \\ Z_{12} \\ \dots \\ Z_{1p} \end{bmatrix} = \begin{bmatrix} X_{11} & X_{21} & \dots & X_{p1} \\ X_{12} & X_{22} & \dots & X_{p2} \\ \dots & \dots & \dots & \dots \\ X_{1n} & X_{2n} & \dots & X_{pn} \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \\ \dots \\ u_{1p} \end{bmatrix}. \quad (20)$$

In this way, the first component is obtained by maximizing its variance, subject to the restriction that the sum of weights u_{1j} squared equals one. Thus, if V is the variance-covariance matrix, then the variance of the first component is:

$$\text{var}(Z_1) = u_1' V u_1. \quad (21)$$

The constraint can be seen as:

$$\sum_{j=1}^p u_{1j}^2 = u_1' u_1 = 1. \quad (22)$$

To obtain the next components is necessary include the orthogonality restriction:

$$\dot{u}_i u_1 = \dot{u}_i u_2 = \cdots \dot{u}_i u_{p-1} = 0. \quad (23)$$

Most books on multivariate analysis techniques mention the need to work with data centred on the mean or standardized for principal component analysis, because the analysis with original data could imply that the first component gets biased towards the mean of the data. On the other hand, Gallagher *et al* (2020) argue that, if the data is not centred, the first component will not be the mean of the data, but the result may tend towards it. Correspondingly, when the data are not centred, the first component captures more variance given that the sum of squares is based on the mean. In this paper, given that the coverage, adequacy and sufficiency are on the same scale (between zero and one) and additionally, we seek to have a direct relationship with the average of the data to evaluate the generic situation, it was decided to work with the variables without any transformation.

Clustering

The last topic to review is clustering analysis, which has the purpose of creating groups of homogeneous observations and at the same time groups as different as possible from each other. The main idea is based on establishing a measure, commonly known as distance, which indicates the degree to which each pair of observations is related. In this case, it is essential to standardize the observations since distance measurements are very sensitive to units.

Despite the different grouping methods that exist, Ward's method was selected given that it proposes all possible combinations for the number of groups considered in each stage and is not based on the distance of the clusters to form the groups. Ward's method calculates the centroids of the groups of the possible mergers and calculates the squared Euclidean distance of all the observations of the group so that in the solution, a lower sum of squares is obtained, and maximum homogeneity is guaranteed. Squared Euclidean distance is defined as:

$$D_{ij} = \sum_{p=1}^k (x_{ip} - x_{jp})^2. \quad (24)$$

When all distances are calculated, the closest observations are grouped and subsequently, this group is replaced by an observation that represents them and that takes the average values of the set of observations that make up the group. This process is repeated until the determined number of groups is reached. It must be noted that this last technique was incorporated to give more interpretation to the results obtained in principal components analysis.